

On the nature of the electron and other particles

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(Dated: November 21, 2008)

An attempt is made to describe the electron as a purely electromagnetic particle. Electromagnetism is reformulated in an algebra which parallels, as closely as possible, the commutation, rotation and Lorentz transformation properties of space-time. Within this framework a minor extension to the Maxwell theory is proposed. An invariant scalar term introduces forces strong enough to confine the electron charge and provides a basis for rest-mass terms. A model for the electron in this framework is presented, the possible nature of other particles is discussed briefly and some possible experimental tests of the model are proposed.

I. INTRODUCTION

This paper considers the underlying nature of material particles in general and of the electron in particular. It arises from a speculative line of reasoning followed by the author and Dr. Martin van der Mark over the last 15 years. It builds upon and expands on a purely electromagnetic model of the electron we proposed some years ago[1–3]. The paper is structured as follows. It is argued that the experimental evidence suggests that the electron must be a purely electromagnetic particle. A minimal modification to the Maxwell equations is proposed. This allows the fields to couple, and leads to circulating solutions with a radial electric and an axial magnetic field, which are identified with the electron and other particles.

In exploring the underlying nature of the electron it is crucial to consider the experimental evidence. An electron positron pair in the spin zero configuration decays to two, and in the spin 1 configuration to three, photons. Two photons, of sufficiently high energy, in a spin zero configuration, can yield an electron positron pair. As seen from this perspective, it would seem blindingly obvious that both are made of the same material, whatever that may be.

Unfortunately none of the current theories have so far proven equal to the task of describing the detailed evolution of a state containing fields alone, to one containing particles or vice-versa. Quantum electrodynamics, which perhaps comes closest, assumes the existence of physical charges and must renormalise its parameters to the observed properties of these charges.

What is required is a theory which exposes the underlying nature of these charges, and can be used as a new basis for electrodynamics, without the necessity for a renormalisation scheme. Such a theory must also be consistent with relativity, with the observed particle spectrum in the standard model, and should lead naturally to a proper basis for quantum mechanics.

In the present work it is speculated that that material particles may best be described by a simple extension of

the theory of continuous electromagnetism. Such an extension allows for (re) circulating solutions corresponding to continuous loops of field. The simplest of these, a simple electromagnetic vortex, corresponds to the electron or positron, with more strongly looped configurations corresponding to the muon and tauon.

Perhaps one of the major barriers to progress is that the experimental evidence seems, on the face of it, contradictory. For example, on the simple matter of the electron size it appears smaller than attometres in deep inelastic lepton scattering[4], has a Compton wavelength of picometres, and yet may be measured to be hundreds of nanometres in size in solid state physics experiments[7].

Theory is no better. Energy considerations mean the electron cannot be smaller than the classical electron radius, a few femtometres, yet many theories regard it as having an essentially zero extent: a point particle. Still others assign wavelengths which make the largest electron in an object twice the size of the object (Schroedinger). It seems that no matter which size you associate with an electron somebody somewhere is going to think you are wrong. What is required is a theory that allows it to appear big in one context, yet small in another[8]. It is to be hoped that the present paper will go at least some of the way to explaining how this might be possible.

Usually one considers the field to be a limit of an average quantity over many quanta (photons)[5]. That this is substantially correct for interactions between charged particles is evidenced by the remarkable accuracy of the predictions of quantum electrodynamics. Here, however, we will consider that the field, considered as microscopically continuous, may itself be the source of quantised sources (charged particles). That is the hierarchy envisaged is that continuous electromagnetism gives rise (only) to quantised vortices with many of the properties of physical particles and it is speculated further that these vortices may interact amongst each other only via the exchange of quantised electromagnetic fields (photons), although this process is not addressed by the present paper and remains to be proven[6].

Some years ago we published a paper on a simple semi-classical model of a circulating photon [1] which had several features in common with the electron such as charge, half-integral spin and an effective size which scaled ap-

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propriately with interaction energy. The main aim of this paper is to extend the scope of the model on the basis of a re-formulation of electromagnetism within that algebra deemed most appropriate for a description of the electron a Dirac algebra [9]. The algebra chosen ensures that all elements, to all orders, transform and commute in themselves and amongst each other, as they should under general Lorentz rotations, Lorentz boosts, rotations of rotations, inversions, reflections, products, quotients and, most importantly, differentials.

The algebra has been chosen specifically as that algebra which most closely mimics the behaviour of space-time in every respect. In particular, the vector differential is a frame independent proper (covariant) derivative. Though a particular frame may be picked for the derivative, this would refer to a particular observer. Since the underlying abstract algebra is frame-independent, there exists a corresponding derivative in every proper instantaneous frame in any conformal geometry. In particular, there exists a corresponding derivative in the frame of the element under study, even if it is lightspeed[10]. The appropriate derivative will depend on the geometry and in complex objects the same derivative may not be appropriate throughout the object, so the algebra must be frame independent and abstract. A translation need be made to a particular frame only if desired and is usually then with reference to some observation. If a better algebra can be found, it is speculated that a re-formulation of classical physics within it may serve to further deepen our understanding of the underlying physical world.

Since it is a little cumbersome to ascribe a separate “observer” to each electron in the universe at each point in phase, the parallel of the derivative which generates the electron dynamics in reality, whatever it is, will be referred to as the “actor”. This allows a clear distinction to be made between the properties of the process which creates or maintains a particle, and the act of observing that particle. Though based on proper relativistic 4-space, the various interesting components such as the fields, currents and invariant scalar prove to have a relationship amongst each other that is more complex than merely 4-dimensional, although it appears in several respects simpler. In particular, the components reduce to four sets of (superimposed) three component spaces, with four extra components (amongst which time and the gauge) through which they interact. This leads to a physical space of the fields which appears three, rather than four-dimensional. The simple appearance, of course, belies the underlying complexity.

The structure of the paper is as follows. We write Maxwell’s equations using a Dirac γ -matrix algebra. This has the advantage that it proves possible to derive all of the Maxwell equations at once without the need to consider the homogeneous and inhomogeneous equations separately, as is usually the case in the more standard approach [5]. That component which will prove responsible for confining electromagnetism, the invariant scalar, will be introduced. In the context of the algebra, we will go

on to discuss the geometrical relationships of the Electric and Magnetic fields, the Currents and the Angular Momenta. The introduction of the invariant scalar is shown to provide a confining component of momentum density. A consideration of particle/antiparticle pair creation leads to a model of a circulating single-wavelength photon vortex, which we identify with the electron/positron and discuss its projections of field, phase and space-time. The possible form of other particles will be discussed briefly.

II. THE ALGEBRA

A 4-vector A is written

$$A = (A_0, \vec{A}) = A_\mu \gamma_\mu = A_0 \gamma_0 + A_1 \gamma_1 + A_2 \gamma_2 + A_3 \gamma_3 \quad (1)$$

with the A_μ being real coefficients and the γ_μ being unit vectors in time and three perpendicular spatial directions. For Cartesian space a useful representation of the γ_μ is given by Dirac matrices, although the algebra will be kept abstract as a proper frame will prove difficult to define for rotating lightspeed objects. As usual, summation is implied for repeated Greek indices which run from 0 to 3. Latin indices will be used to indicate a three-component object, for example the spatial part of the four vector or the magnetic field, and run from 1 to 3.

Three vectors are distinguished by an overarrow, as in Eq. (1) with the dot and cross symbols referring to the usual 3-vector products.

The definition of the basis elements includes the properties of the metric tensor $g^{\mu\nu} = g_{\mu\nu} = \text{diag}(+ \ - \ - \ -)$ and corresponds to the Lorentz metric:

$$\gamma_0^2 = -\gamma_i^2 = 1 \quad , \quad i = \{1 \dots 3\} \quad (2)$$

Amongst other things, this means that A^2 gives precisely a Lorentz-invariant scalar product

$$A^2 = A_\mu A_\nu \gamma_\mu \gamma_\nu = A_0^2 \gamma_0^2 + A_1^2 \gamma_1^2 + A_2^2 \gamma_2^2 + A_3^2 \gamma_3^2 \\ = A_0^2 - A_1^2 - A_2^2 - A_3^2 \quad (3)$$

and the proper invariant interval ds of space-time

$$(ds)^2 = (dx_0)^2 - (dx_1)^2 - (dx_2)^2 - (dx_3)^2 \quad (4)$$

where ds is both real and time-like (γ_0) for subluminal world lines.

For the quotients $\gamma^\mu = 1/\gamma_\mu$ we have

$$\gamma^0 = \gamma_0, \quad \gamma^i = -\gamma_i \quad (5)$$

As a consequence of the quotient, the spatial part of the four-vector derivative in the Clifford-Dirac algebra has opposite sign to the vector

$$d = \frac{\partial}{\partial x_\mu \gamma_\mu} = \partial_\mu / \gamma_\mu = \gamma^\mu \partial_\mu = g^{\mu\nu} \gamma_\mu \partial_\nu = \\ \partial_0 \gamma_0 - \partial_1 \gamma_1 - \partial_2 \gamma_2 - \partial_3 \gamma_3 = \gamma_0 \partial_0 - \left(\begin{array}{c} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{array} \right) \vec{\nabla} \quad (6)$$

In this sense, the γ^μ correspond to the covariant basis vectors and we write them here and only here, with an upper index for this reason. The point that we do not need to (and in fact would not want to) elsewhere is crucial and perhaps deserves some further explanation for those not familiar with this kind of algebra. Because the 4-vector differential operator contains (inverse) Dirac elements explicitly means that it transforms properly in the relativistic sense. It deals correctly not only with the scalar product, as does the more conventional approach, but also with the non-commutative rotations and boosts and angular momentum-like objects on which it operates. At the same time, it contains the proper 4-dimensional scaling properties of space and time. The differential is a special kind of inverse vector, and these scale relativistically, with the result inheriting the proper transformation properties in the frame of the derivative. For a given lab frame the variation it describes is with comparison to the ruler-clock of the observer. It is a full covariant derivative. How important this is will become clear in the unique way that it gives rise to all of the Maxwell equations at once, with all the right signs, and nothing more, in the next section. It is, in a real sense, the key to the power of the algebra.

Products or quotients amongst the vector basis elements γ_μ form linearly independent elements of higher or lower grade. There are 6 independent terms of the form $\gamma_\mu\gamma_\nu$ which we abbreviate with $\gamma_{\mu\nu}$, the bivector basis elements, representing unit planes. Just as the γ_i form a basis for translations in 4-space, the higher grade elements γ_{jk} form the basis elements of rotations and the γ_{i0} the basis elements of boosts (Lorentz transformations) in the (three component) spaces which they span. Note that $\gamma_{\mu\nu} = -\gamma_{\nu\mu}$ for $\mu \neq \nu$; any exchange of adjacent indices generates a factor of minus one.

There are four independent trivectors (the pseudo 4-vector basis elements, representing unit volumes) of the form $\gamma_\lambda\gamma_\mu\gamma_\nu = \gamma_{\lambda\mu\nu}$. these may also be taken as a basis for the complete algebra if desired. There is also a single independent quadrivector γ_{0123} , the pseudoscalar, representing a unit hypervolume. This, together with the generator basis vectors γ_μ and the invariant scalar $\gamma_0^2 = 1$, leads to 16 linearly independent unit elements which, together with their counterparts with negative sign, form an algebraic group of 32 elements. The real algebra with this group requires only a positive 16 unit basis because the minus sign may be absorbed in the real coefficients. Choices here, however, influence the handedness of the (four) three component sets and must be made with care.

So called multivectors can be formed using these elements. The most general multivector $\Psi = s + v + b + r +$

$t + q$ is defined as

$$\Psi = s + \gamma_0 v_0 + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \vec{v} + \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} \vec{b} + \begin{pmatrix} \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} \vec{r} + \begin{pmatrix} \gamma_{023} \\ \gamma_{031} \\ \gamma_{012} \end{pmatrix} \vec{t} + \gamma_{123} t_0 + \gamma_{0123} q \quad (7)$$

The three-component and one-component objects are made explicit here using a column vector notation. A short calculation shows that $\gamma_{i0}^2 = \gamma_{123}^2 = 1$, $\gamma_i^2 = \gamma_{ij}^2 = \gamma_{0ij}^2 = \gamma_{0123}^2 = -1$. Of the 10 elements which square to -1 , not one commutes with all other elements, that is, none behave like the complex number $\mathbf{i} = \sqrt{-1}$. For the even subalgebra $\{1, \gamma_{i0}, \gamma_{jk}, \gamma_{0123}\}$, however, the quadrivector γ_{0123} , because it commutes with all even elements may take the role of the unit imaginary number $\sqrt{-1}$ for this subset. The sixteen-element set generated from the basis γ_μ on the Lorentz metric $(+ - - -)$, forms a geometric Dirac algebra, the Clifford algebra of space time $\mathcal{C}\ell_{1,3}$.

III. THE MAXWELL EQUATIONS

Starting with a four potential $A(x)$ defined over all space-time x and using the algebra with the vector differential introduced above the Maxwell equations may be derived in a particularly beautiful and compact way. In what follows ϵ_0 , \hbar and c are set equal to 1 but a specific translation to S.I. units will be made where appropriate.

Let the four potential be $A = (A_0(t, \vec{x}), \vec{A}(t, \vec{x}))$ with A_0 the scalar potential and \vec{A} the vector potential. In accordance with the previous section, we write

$$A = A_\mu \gamma_\mu = A_0 \gamma_0 + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \vec{A} \quad (8)$$

It will not be a surprise that the full multiplication of the differential (inverse 4-vector), with a general 4-vector leads to patterns which would be familiar to those who developed electromagnetism in a 3-dimensional space in the nineteenth century. Maxwell's equations are, and always were, fully relativistically covariant. To make contact with the familiar, we write this in terms of the more familiar 3-space forms such as the three-vector potential \vec{A} , the electric field \vec{E} and magnetic field \vec{B} and the standard (three-dimensional) dot and cross product, whilst the full 4-space algebra is maintained by means of the positional column notation introduced above for the proper components. With these conventions we may write the

16 ($= 1 + 3 + 3 \cdot 2 + 3 \cdot 2$) terms of the full product dA as

$$dA = \partial_0 A_0 + \vec{\nabla} \cdot \vec{A} - \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} (\partial_0 \vec{A} + \vec{\nabla} A_0) - \begin{pmatrix} \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} \vec{\nabla} \times \vec{A} \quad (9)$$

which is the sum of a scalar part P and a bivector part F , so we can write $dA = P + F$, with

$$P = d \cdot A = \partial_0 A_0 + \vec{\nabla} \cdot \vec{A} \quad (10)$$

The scalar P is related in some ways to the conventional gauge but because of the context of the Dirac algebra in which it is embedded here it is much more general and far more potent. P is invariant under a Lorentz transformation. In the extended electromagnetism introduced later it introduces a ponderous mass and acts as a pivot about which light may turn, creating a photon vortex. It is also the “direction” of the energy component of a four-momentum. In order to distinguish it, we will call it the invariant scalar or simply the pivot. There is also a possibility to use its dual, the quadrivector, which is also invariant under a Lorentz boost, as an additional gauge-like term but that freedom will not be exploited here. For an arbitrary scalar function Λ the gauge freedom is expressed by the transformation $A \rightarrow A + d\Lambda$ and hence $P \rightarrow P + d^2\Lambda$. Setting $P = 0$ (for all coordinates) includes then a strong form of the Lorenz gauge condition (setting its derivative zero is sufficient to define the Lorenz gauge itself).

In Eq. (9) the electric field $\vec{E} = -\partial_0 \vec{A} - \vec{\nabla} A_0$ and the magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$, may be identified in the usual way. Together these terms form a bivector denoted by F and correspond to the antisymmetric Faraday or field-strength tensor $F^{\mu\nu}$ [5]:

$$F = \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} \vec{E} - \begin{pmatrix} \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} \vec{B} \quad (11)$$

This is more similar to the spinor form [11, 12], though the space at hand is larger than a spinor space. Properly, in Eq. (11) the electric and magnetic fields have a bivector form γ_{i0} and γ_{ij} respectively and appear neither as polar or as axial vectors nor simply as a set of tensor components. It is easy to show that these components transform properly under Lorentz rotations and boosts [16]. For example, the perpendicular field components transform under a boost, whereas the parallel components are unaltered, in contradistinction to the case of the 3 spatial components of a 4-vector. As mentioned above, the proper relativistic and commutative transformation properties are crucial in maintaining the simple unified form of all of Maxwell’s equations in what follows.

Consider the dynamics of dA . Usually at this point, a 4-vector current source term J is introduced:

$$d(dA) = d(P + F) = dP + dF = J \quad (12)$$

Using $P = 0$ at all coordinates, so that $dP = 0$, we get

$$dF = J \quad (13)$$

which, under multiplication and expansion of the 24 terms on the left, gives

$$\gamma_0 \vec{\nabla} \cdot \vec{E} = \gamma_0 J_0 \quad (14)$$

$$\gamma_{123} \vec{\nabla} \cdot \vec{B} = \gamma_{123} J_0^m = 0 \quad (15)$$

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} (\vec{\nabla} \times \vec{B} - \partial_0 \vec{E}) = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \vec{J} \quad (16)$$

$$- \begin{pmatrix} \gamma_{023} \\ \gamma_{031} \\ \gamma_{012} \end{pmatrix} (\vec{\nabla} \times \vec{E} + \partial_0 \vec{B}) = \begin{pmatrix} \gamma_{023} \\ \gamma_{031} \\ \gamma_{012} \end{pmatrix} \vec{J}^m = 0 \quad (17)$$

Which are immediately recognisable as the full set of Maxwell equations in natural units, with all the correct signs, though with the proper multivector form of the equations within the algebra made explicit.

This may seem, in the first instance, to afford little progress over the conventional 3-space analysis or the conventional tensor approach however it has one crucial advantage: retaining *all* products within the algebra, both the homogeneous Eqs. (15) and (17) and inhomogeneous Eqs. (14) and (16) are contained in the single equation, Eq. (13), without the need to introduce a separate dual field strength. This is in contrast to other developments using either standard notation [5] or the algebra of forms [17], where in both approaches there are two quantities representing the same physical fields. By carrying every part of the product the full Maxwell equations are obtained, with all the correct signs and nothing more.

Another possibility is to use the invariant scalar itself to introduce a J [13]. Taking $dP = -J$ such that $d^2A = 0$, leads once again to $dF = J$ at this point. Either way, note that Lorentz covariance is broken by the introduction of a source current term. Only for $dP = 0$ is A a true 4-vector for which the components A_0 and \vec{A} transform appropriately under a Lorentz transformation [5, 14, 15]. This is as it should be: charge is invariant under a Lorentz transformation and should not transform as a 4-vector. This is related to the fact that the Helmholtz decomposition to a vector potential requires a solenoidal (divergence free) field: not the best condition to impose if one wishes to investigate the origin of charge. In particular, this means that for the scalar to generate charge it must arise from some multivector component other than the vector. How and why this might arise will be treated in more depth later in this paper.

Note also that a general current cannot be represented by the 4-derivative of a scalar alone and this is a potential difficulty with introducing the charge in this way. The full current, however, also contains terms originating from the differentials of the fields (24 terms). These are quite general and may remove such barriers in future

work. The fields also do not transform as a 4-vector (and, with six components, of course they should not). Given the algebra in which they are expressed, it should prove no surprise that they transform properly relativistically as field components[16].

In the past there has been some discussion as to whether a proper description of electromagnetism should start with the fields, or with the vector potential. The present analysis reveals the paucity of either approach. Both the four components of the vector potential as well as the six components of the field and an 11th element, the invariant scalar, are required at the very least. In addition, in what follows at least three of the four components of angular momentum will prove interesting, as must eventually be the case if electromagnetism is ever going to include quantum spin.

Note that in Eq. (12) the product was written $d(dA)$ and not in the simpler (wave equation) form d^2A . The reason for this is that the “ ds ” refer in general (and in fact usually) to different frames of reference. For a measurement of something in the frame of observer “ o ” measured using their own ruler clock (corresponding to an eigenderivative d_o) when acted on by a field produced by an actor derivative d_a elsewhere and elsewhen (the eigenderivative in the field-creation process), one should properly distinguish these as $d_o(d_a A)$. For example, when an observer on the beach feels the warmth of the sun on their face, the actor derivative and the vector potential producing the (photon) field were on the sun and the observer on earth roughly eight minutes later. The derivative is not a passive thing either in reality or in the current paper. It is a (meagre) attempt to describe the dynamical process each particle does for itself naturally all of the time (without using calculus!).

Having a single equation gives hope that a further extension of electromagnetism beyond the Maxwell equations may give fruitful results. If one looks at the generalised Lorentz force, the products of fields and their differentials, one has 1024 terms starting from the differentials and potentials (not adequate by itself, since the potentials alone cannot generate a charge, as discussed above one needs interactions), or 196 starting from the differentials, the invariant scalar and the fields. Many of these cancel but which ones cancel depends on the choice of sign at each stage in the product. If the underlying guidance provided by the algebra does not properly parallel reality, one has little hope of getting through the jungle.

IV. THE GEOMETRICAL RELATIONSHIPS OF FIELDS, CURRENTS AND ANGULAR MOMENTA

While the Maxwell equations presented in the last section may be familiar, the multivector form in which they appear in the present form may be less so. These forms will be discussed in some length, as they are crucial to un-

derstanding in which “space” the field distributions which follow are drawn. In particular, they will *not* be drawn in the vector part of a 4-vector space. The figures presented later will be multivector-dimensional and it is the purpose of this section to explain what is meant by this. This point has caused some confusion in the interpretation of earlier work[1]. The homogeneous Eqs. (15) and (17) are 4-vectors and the inhomogeneous Eqs. (14) and (16) are 4-trivectors.

Even if one divides out the multivector forms from each side, one is still left with the bivector forms of the fields themselves: the Magnetic field appears in a form similar to a three-dimensional axial vector with two spatial indices, however the Electric field does not appear as a vector but as a space time bivector with one space and one time index. This betrays a fundamental confusion about the nature of the Electric field, which is perhaps more widespread than it should be. It is not a vector, it does not transform as a vector, it is not even the spatial part of some four-vector. It, in any formalism, has only three components. Together with the three components of the magnetic field these transform as an anti-symmetric tensor. The potential confusion arises, in part, because of our deep familiarity with the 3-vector formalism in which we all first learned electromagnetism. The proper form of the equations of electromagnetism is given by equations Eqs. (14),(15), (16) and (17), irrespective of any interpretation we may give them. Whether we choose to visualise the various fields as 3-space vectors or keep them in tensor or bivector form is a question of custom and taste. Their transformation properties amongst each other, however, are not and are uniquely fixed by the nature of the universe. These are best treated using an algebra which parallels those properties as closely as possible.

Though the field is derived from a 4-vector potential through Eq. (9) it is, of course, not itself a 4-vector. It has six components which we, conventionally, choose to split into two sets of three. These six components are all linearly independent of each other and linearly independent of the 4-vector components as well. They are in the direction of a subset (the bivector subset of unit oriented planes) of the sixteen linearly independent basis elements described above. Historically, the familiar 3-space polar vectors \vec{A} and \vec{E} and the 3-space axial vector \vec{B} have been derived as projections of the 4-potential A and fields F on 3-dimensional space using the following rules respectively:

$$\gamma_i \rightarrow \hat{x}_i \quad (\text{polar vector}) \quad (18)$$

$$\gamma_{0i} \rightarrow \hat{x}_i \quad (\text{polar vector}) \quad (19)$$

$$\gamma_{jk} \rightarrow \hat{x}_i \quad (\text{axial vector}) \quad (20)$$

Where, in these expressions, different Roman letters correspond to different indices. The first of these projections involves a Lorentz scaling factor. The others are more complicated, including both a scaling factor and an implicit projection. The second projects out time,

the third projects to a perpendicular vector in a particular Lorentz frame. These projections are useful aids to thinking about geometry in many circumstances but it is important to bear in mind that, in rotating four-dimensional lightspeed objects in particular, the notation of the projection onto a perpendicular vector has limited visualisation value since all such vectors will be compressed into a plane for an external observer. For a discussion on the problems of correlation of three and four-dimensional forms see, for example [18]. Note that Eq. (20) corresponds to the Hodge dual in 3-space [19] which links the cross product of two vectors \vec{a} and \vec{b} (which projects onto a vector \vec{c}) with the 3-space outer product: $\vec{a} \times \vec{b} = \gamma_{321}(\vec{a} \wedge \vec{b}) = \vec{c}$. The three distinct projections in Eqs. (18)–(20) behave similarly to each other in many respects, in particular they behave in the same way under translations, though there are some important differences. For example, \vec{A} and \vec{B} behave differently under reflections and \vec{A} and \vec{E} behave differently under time reversal. Although one can treat \vec{E} , \vec{B} and \vec{A} as 3-space vectors (and draw them!), it is important to keep firmly in mind that they are all in fact distinct linearly independent quantities. The only proper way to keep track of all of these properties is to use the Dirac-Clifford algebra in products and quotients, and translate back to the familiar 3-space forms afterwards if desired.

So far the projection properties of only nine of the sixteen linearly independent basis elements appropriate to the model to be presented have been discussed. For completeness, the others are described briefly. Of the remaining seven three are in the direction of the angular momentum $\vec{\mathcal{J}}$ which is also a projection on 3-dimensional space but now of some trivector onto its dual

$$\gamma_{0jk} \rightarrow \hat{x}_i \quad (\text{axial vector}) \quad (21)$$

Note that this involves both an implicit time projection and a projection to a perpendicular vector as well as a Lorentz scaling factor. The remaining four are: the invariant scalar, the unit vector of time γ_0 , the corresponding trivector γ_{321} and the quadrivector γ_{0123} . Here, there is not a history of projecting these onto each other with, in particular, the time and the scalar being treated quite separately. The remaining two may, however, be taken, in the same sense as above, as projecting onto either the time unit vector, or onto the scalar but again they may have different properties under the various Poincaré group transformations. For example, the scalar and quadrivector are invariant under a Lorentz boost, whereas γ_0 and γ_{321} transform as the time component of a four-vector.

Viewed in the light of these projections, the world we live in is rather more complicated than merely four-dimensional, though, in some senses, it appears simpler. There are sixteen basis elements all of which may play a role. The transformations of the vector itself are exactly the familiar ones of Minkowsky space-time. This in itself, however, does not account for all of the complex-

ity of the world around us. As an example of just how serious an implicit simplification is made consider, for example, the 3-dimensional structure of a crystal. The macroscopic structure is governed by interatomic forces. The electric field structure in the crystal is spanned by the basis $\gamma_{01}, \gamma_{02}, \gamma_{03}$, a basis with three, not four degrees of freedom. At the same time magnetic forces may also play a role spanned by the (conventionally axial vector) basis $\gamma_{12}, \gamma_{23}, \gamma_{31}$, as indeed may the spin spanned by the basis $\gamma_{012}, \gamma_{023}, \gamma_{031}$.

From the present perspective, there is a sense in which the crystal has four three-dimensional sets of properties which are superimposed, referenced to some point in time, onto points on the same three-dimensional spatial grid (in the rest frame of the crystal), with the implicit adoption of the projection rules of Eqs. (18)–(21). The properties under translation of the four groups of three elements described in the projection relations are the same and this justifies such a mapping but the way they behave under other transformations such as rotations and boosts is different. A fuller discussion of these properties may be found in a recent thesis [16]. Also the properties of the remaining four elements, amongst them time itself, play an important role in the measurement process. In this sense, the world grid in which we live is revealed as four superimposed three-dimensional frameworks, with additional features provided by four more one-dimensional degrees of freedom. Any measurement of any properties of the crystal such as its vibration, magnetisation, or emission of light will be carried out in terms of the four-dimensional set of space and time in the frame of the observer which will in general be different to that of the crystal (the actor frame) and perhaps expressed as derivatives with respect to space and time in that frame. In a proper sense the 4-D space of the observer is with reference to her own derivative set. Because the derivatives are inverse vectors, they modify the form, symmetry and relativistic properties of what may be observed. A simple example has been given above in the Maxwell equations, where the four derivative of the four vector potential gives rise to seven linearly independent components (one scalar and six fields). Again, this reveals the paucity of using the 4-vector alone to describe the whole of electromagnetism. They neglect the observer/actor frame in which the differentials are defined.

At this point, it is worth reiterating that, although we have sixteen linearly independent basis elements arranged in groups of three and one, we are not talking about some obscure sixteen-dimensional space: such things as the basis directions of the electric field arise through the properties of the four-dimensional vector basis under multiplication and division. A familiar analogy is the linear independence of the three (axial vector) axes of rotation in 3-dimensional space from the three vector dimensions themselves. Rotations are linearly independent of translations, and hence give three extra “dimensions” to movement and yet we are not in the habit of saying we live in a seven-dimensional space-rotation-time

(and do not need to be). With the four-dimensional algebra used here, in addition to the unit elements corresponding to rotations, we have unit elements corresponding to such things as Lorentz boosts and to angular momenta as well. For the reader unfamiliar with such properties there is an excellent introduction to them by the Cambridge group [20] and much information may also be found on their website.

In summary, the sixteen basis elements of the algebra may be grouped in two ways. Grouped by order there are one scalar, four vector, six bivector, four trivector and one quadrivector components. These represent unit points, lines planes, volumes and hypervolumes in spacetime. Grouped according to their historical 3D projections there are four 3-vector sets projected onto the direction of the vector, the electric field, the magnetic field, the angular momentum density and the vector itself. There are also four other distinct elements, including the invariant scalar and the time. Which grouping one chooses does not matter, provided one always uses the correct transformation properties for each element derived from the four vector basis elements, using the full mathematical formalism outlined above. In particular, we must not fall into the trap of thinking that, because we are describing something as a vector, it will necessarily transform under reflections, rotations and boosts in the same way as a vector. In fact, none of the sets of conventional component objects, even the spatial part of the 4-vector, transforms in every respect as a 3-vector[16].

V. ELECTRON-POSITRON PAIR CREATION

The tools necessary to understand the continuous process of electron-positron pair creation within electromagnetism are now in place. The simplest framework required to find the form of the solution is through the energy and momentum density of the field and the invariant scalar (pivot) term. The work will be carried out in terms of energy (scalar) and momentum (space-time bivector) and not in the more conventional terms of energy-momentum 4-vector. It is a relatively simple matter to translate a posteriori to the 4-vector form, if desired.

The Hermitian conjugate within the algebra corresponds to reversing the sign of all components which square to -1. For the fields this reverses the sign of the magnetic field but leaves the electric field unchanged, corresponding to a field with reversed phase development, a counter-propagating field.

Consider a detailed evolution of a field F corresponding to one incident photon and a counter-propagating field F^\dagger corresponding to a photon travelling in the opposite direction. Ignoring for a moment the quantised nature of the incident fields and concentrating purely on the electromagnetic aspects, the initial fields are well described by the free space Maxwell equations in the Lorenz gauge ($P = 0$) with charge zero everywhere. If F de-

scribes a right-handed field propagating leftwards, then the phase-reversed field F^\dagger describes a right handed field distribution propagating rightwards. As the fields overlap they form a curious distribution well-known to the laser physicist. This is the so-called twisted mode structure. For a fuller description of this the reader is referred to earlier work[1]. The overlapping fields have \mathbf{E} parallel to \mathbf{B} everywhere (and for all time). The result is that the Poynting vector $\vec{E} \times \vec{B}$ is everywhere zero and in this sense the field distribution is non-propagating.

If this were literally the case, one might imagine a situation where more and more of the incident electromagnetic wave train piled up into the overlapping volume and added to the energy density in that volume. At low energy, of course, the overlap would never be perfect and the situation would quickly revert to two photons again. The configuration, however, corresponds to that leading to particle pair creation at sufficiently high energy, where the two incident photons are initially in a spin zero state. This state has both photons right or left circularly polarised (right and left have opposite spins but direction reversal also reverses the spin). Conventionally, in classical electromagnetism, one assumes that the fields interact only with charges. Self-evidently, as the energy of the incident photons increases, this must break down at some point, since initial field-only distributions evolve into a pair of charged particles in the pair creation process. Labelling one field F and its counter-propagating partner, the Hermitian conjugate field, F^\dagger the energy-momentum density in the fields is

$$M_{field} = \frac{1}{2}(FF^\dagger) = \frac{1}{2}(\vec{E}^2 + \vec{B}^2) + \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} (\vec{E} \times \vec{B}) \quad (22)$$

Which is the conventional expression for the energy-momentum density of the electromagnetic field but again with the proper multivector form of energy (scalar) and momentum (space-time bivector) made explicit. The multivector combination, though it is not itself a four-vector, may be transformed to one by multiplying or dividing by a unit time in the observer/actor frame. Such a product is a generalised electromagnetic energy density.

In some respects the expression above has a similar form to the conventional product in quantum mechanics representing the energy density $\psi^* \psi$. The scalar term is just the energy-momentum density but the full product contains higher order terms as well. Equations of motion for such a state may be obtained by setting its four-derivative to zero, that is: $d(FF^\dagger) = 0$. Such an equation is a kind of generalised Hamiltonian equation carrying higher order terms (momentum density) from the initial field product as well as the scalar (energy density) terms. By itself such an equation can represent only the initial photons. To move to a description of particles one must introduce a coupling term and this is most conveniently provided by the the invariant scalar pivot.

Introducing the invariant scalar pivot term P into the energy momentum density gives:

$$M_{field} = \frac{1}{2}(F + P)(F^\dagger + P^\dagger) = \frac{1}{2}(\vec{E}^2 + \vec{B}^2 + P^2) + \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} (\vec{E} \times \vec{B} + P\vec{E}) \quad (23)$$

As can be seen, the invariant scalar adds terms both to the energy density and to the momentum density. It is these new terms which are the key to understanding how rectilinear photon propagation in the initial state may be transferred to rotational, vortex-like solutions to the final state corresponding to an electron-positron pair. The extra term in the momentum $P\vec{E}$ is always perpendicular to the Poynting vector. Hence in a radial electric field the derivative of this acts as a central force. It is this central force which, it is speculated, allows electromagnetism to confine itself in particle-like solutions. That is this term constitutes (at least one component of) the Poincaré stresses which confine the electron. This is the most important result of the paper.

The scalar term, being invariant under a Lorentz transformation, certainly has the correct transformation properties for the job. For a radial electric field the $P\vec{E}$ term represents an inward-directed momentum component in Eq. (23), that is it tends to direct the flow of electromagnetic energy inwards, in the same way as a central field of force creates an inward directed momentum component. The circulation in turn reconfigures the photon fields to give a charge[1]. Since the phrase “source term” is often used to denote the charge itself, in the present approach the invariant scalar is in some sense the source of the “source term”.

All that is required for a circulating, vortex-like solution in a radial electric field is that the invariant scalar component P be sufficiently strong. From whence might this “sufficiently strong” component arise? It could either be carried by the initial photons or created in the interaction process. Let us deal firstly with the former. Setting $P = 0$ is a strong form of the Lorenz gauge, (which requires only that $dP = 0$). Is it possible then to add a constant term to the initial Photon fields? Under a Lorentz transformation, the fields transform as they should but the scalar term is invariant. In Eq. (23) the invariant scalar term leads to an extra contribution to the energy. This would be fine for a particle and could be identified with a (rest) mass but it is not fine for a photon which must transform as a field, hence the initial (two photon) state must be described by having zero pivot, $P = 0$. That is electromagnetism, as it stands, is precisely characterised by having zero pivot.

Given that the scalar component is not carried by the incident photons, it must be created in the interaction process. A possible origin is the energy density in the fields. This is described conventionally by (using S.I. units here) $W = \frac{1}{2}(\epsilon_0\vec{E}^2 + \epsilon_0c^2\vec{B}^2)$. This is the same as

the first term in Eq. (22). That is an energy (or mass) density is ascribed to the square of the field strengths, times the relevant normalisation constants. This, however, is mere maths. The field is still a field, though it has a potential if destroyed or confined of yielding that energy over an appropriate volume measured in some relevant frame of reference. Note that the frame is crucial: that the proper frame for the photon is a lightspeed frame where, mathematically, the field energy density for an on-shell photon is always infinite, not because of an infinite photon energy but because of an infinitesimal extent in the direction of photon travel due to the Lorentz contraction, yielding an infinite energy density. This is not to say that these infinities need exist in nature; real photons are always ever so slightly off mass-shell. Further, the observer frame is always anchored to something with a good deal of rest mass, making all measurement volumes comfortably finite.

In any event, we are left with the second possibility, that some of the initial field is converted to a scalar component which constitutes the P required for photon confinement in the interaction process. In the twisted mode two components contribute to each of the Electric and Magnetic field components. These are always parallel to each other but vary in strength periodically reaching a maximum where the fields are parallel. At this point, the dual field is antiparallel, that is a maximum in the electric field corresponds to a minimum in the magnetic and vice versa. Cancellation occurs for antiparallel components of the initial fields. There is an asymmetry here between the electric and magnetic field. The product of two equal antiparallel electric fields yields a negative result $\gamma_{i0}\vec{E} * -\gamma_{i0}\vec{E} = -\vec{E}^2$, whereas that for antiparallel magnetic fields a positive result $\gamma_{ij}\vec{B} * -\gamma_{ij}\vec{B} = +\vec{B}^2$. Since the result for a scalar squared is positive definite, this may suggest that it is the magnetic field which may most readily couple to a scalar component on cancellation. At this point, an analysis necessarily goes slightly beyond Maxwell’s equations as they stand, if only because the creation of an invariant scalar component implies the creation of a component which transforms as a rest mass. Also one removes the possibility of describing the fields as arising from a purely 4-vector potential. This is a high price, but perhaps one worth paying if the prize is an understanding of the Poincaré stresses, the nature of charge of half integral spin and of the exclusion principle.

Note that in Eq. (23) the magnetic terms cancel identically, meaning that a magnetic monopole solution, with the magnetic field radial, would not have a stable solution. The extra term in the energy P^2 , although it appears in the same form as the other two, has a different character. P is invariant under a Lorentz transformation, hence this term acts as a rest mass.

The time derivative includes force terms, and the full derivative generalised force terms similar, though not identical, to those discussed in the last section. Note also that, here, the full machinery of the Dirac algebra

is not required, and the scalar pivot could be introduced in a more conventional formalism in a minimal extension to the Maxwell equations. Where the machinery is required, however, is in understanding what such terms really mean at the point where they are introduced as drawn objects. It will always be possible, once the proper solutions have been found, to re-formulate them in a more conventional way.

At this point, it is worth noting that the generalised force equation $d\frac{1}{2}(F+P)(F^\dagger+P^\dagger)$ has a similar form to the Dirac equation, having invariant scalar (mass terms) arising from the pivot and a proper 4-derivative of an energy momentum. That it has a similar form means that it has similar solutions. The solution space, however, differs in that these are electromagnetic fields in bivector form rather than spinors. Note, however, that the spinor algebra is a sub-algebra of the Clifford-Dirac algebra used here. Note also that these forces are not exchange forces. They are far stronger; strong enough to bind photons, confine the electron charge, and generate the force responsible for the exclusion principle.

In the Dirac equation one does not begin with the fields. The underlying material of which the electron is made is viewed as a sort of scalar mass density, but modified by the properties of the spinors required to obtain the correct commutation relations and to ensure a relativistic equation. The coupling between the field and charge is introduced through an extra contribution to the momentum represented by the product of charge and vector potential $e\vec{A}$, the so-called minimal coupling.

In Eq. (23) above, the corresponding term appears as a product of invariant scalar and field $P\vec{E}$ which, with reference to the invariant scalar form of the Maxwell equations, differs from $e\vec{A}$ by a change in the ordering of the differential operator. That is $P\vec{E}$ constitutes a minimal coupling, but of pure fields and not of fields and charges[13]. That the invariant scalar may introduce some of the features of charge and current was mentioned briefly above. Note again that the cost of introducing charge in this way is that all gauge freedom is used up.

From here it is possible to develop the theory further to include generalised forces and we have carried out much work in this direction[8]. In the present paper the full machinery is not required since the energy momentum transport direction is already defined by Eq. (23) and the form new solutions must take is clear from these. Eq. (23) together with the Maxwell equations, is sufficient to propose a twisted-looped solution at the point of pair-creation. In fact, note that, for non-zero pivot, *only* rotational (and hence quantised) solutions exist.

In the initial state one has one leftwards propagating photon of definite helicity, and one rightwards propagating photon with the same helicity, forming together a spin zero state. These are, at the same time solutions to the Maxwell equations. In the final state one generates two regions with equal and opposite pivot terms. These form two spinning vortices of energy, two particle-like states, localised in space with radial electric fields and (rest)

mass. These are solutions of the extended equations including the invariant scalar term, but not of the original equations. It is these states which are identified, in the simplest such case, with the electron and the positron.

Consider the infinitesimal development of the initial fields. This must be, given the arguments above, that of the free space electromagnetic field modified only by the fact they begin to overlap in a twisted mode structure. In the interaction region, by supposition, two regions with invariant scalars of opposite signs are being created from energy supplied by some of the initial field, the positive scalar the proto-electron and the negative the proto-positron. With reference to the initial fields alone, where $P = 0$ the direction of momentum density is given by the Poynting vector $\vec{S} = \vec{E} \times \vec{B}$. Clearly, since $P = 0$ in the initial state, it should also be zero in the final state. That is $P = 0$ is conserved overall, corresponding to the conservation of charge, but P^2 may not be, corresponding to the creation of rest mass(es). Envisaged is one region of positive P corresponding to the proto-electron, and one of negative P corresponding to the proto-positron.

The momentum density is a bivector of the same form as the electric field, that is a time-space bivector γ_{0i} . Let us consider the phase to be along this bivector direction and the momentum density in the rest frame of the particle pair to constitute a flow of electromagnetic fluid. The bivector momentum is used rather than the vector position as the former is well-defined in the incident state, whereas the latter is not.

As phase progresses, the electric and magnetic field vectors will twist about this direction according to the development equations, similar to the case of a free-space photon but increasingly will also turn modified by the effect of the invariant scalar term. The direction of this modification depends on the sign of the scalar. Note that the torsion of the fields about the propagation direction is not a weak effect. It is enshrined in the Maxwell equations, and hence in the present context is hand-of-god strong. A temporally varying magnetic field necessarily produces a spatially varying electric field in the right orientation and vice-versa. Note that the energy density increases as fourth power of the incident photon energy/momentum. This means the potential pivot term increases an order faster than the momenta, leading, together with the intensification of the electric field to a pivot term $P\vec{E}$ which will clearly become strong enough at some point to overwhelm the incident momentum and force it into a complete loop within a single wavelength. At that point, dual-vortex production, particle pair creation, may take place.

Take the incoming photon directions to lie on the x axis and the electric field in the interaction region to define the positive y direction. In the region of positive P incoming momentum will be directed in the positive y direction (proto-electron) and in the region of negative P in the negative y direction(proto-positron), forming two vortices in the x-y plane.

Clearly, much can be gained by considering the two vortices to be equal and opposite in every respect, giving conservation of momentum, angular momentum, charge and everything else. Some insight into the field configuration at the point of particle creation can be gleaned, however, by considering an infinitesimal element of momentum, constituted by an element of electric and an element of magnetic field, incident on a pre-existing region of invariant scalar sufficiently strong to force it into a loop. We consider the case of the positron (P negative), simply because it is easier to draw a field configuration with the arrows pointing outwards. As phase increases the fields twist about the axis of momentum flow, however this axis itself is deflected in the opposite direction to the electric field (inwards) according to the strength of the $P\vec{E}$ term. The two rotations of the twist and the loop, together with the constraints given by Eq. 22, and maintaining throughout the torsion about the field propagation direction required by the development equations, results in an initial field configuration on pair creation illustrated diagrammatically in Fig.1

The photons and loops are not shown to scale. A single wavelength of the incident photons λ_c is shown, and the loop radius is roughly a twelfth of this ($\lambda_c/4\pi$). An individual loop with P negative, corresponding to the positron (since this is easier to draw) is illustrated in Fig. 2. The electric field arrowheads are denoted by cog-like blades (green), and the magnetic by gear-like truncated hollow cones (blue). The shapes have been chosen primarily to reflect the underlying bivector forms of each field although it is worth noting that this also does due reverence to Maxwell's original conception of electromagnetism. The path of momentum flow, where the fields are defined, is marked by the glass braid. This path corresponds to that of the "twisted strip" discussed in earlier work[1].

This is the field configuration discussed in earlier work for the eye of the torus[1], though in the present work the proper space in which the toroidal form is embedded is more explicit (it is the momentum space spanned by the γ_{i0} and not the vector space usually utilised by human observers). Note that the figure, like reality, superimposes three different spaces. In this sense, it is nine-dimensional plot (which turns out to simplify to four important directions). The magnetic field is an axial vector in γ_{ij} space. In the configuration illustrated it turns out to have a nearly fixed direction in this space (it has a dipole nature) and so may be represented by a single direction (e.g. γ_{12} for the "z" direction). This is the first "direction" albeit expressed in a bivector and not a vector space. The electric field, likewise, has a fixed direction in its space (the γ_{i0} space) but this is *radial* corresponding to a spherically symmetric electric field. This is the second direction. The momentum executes a loop in a γ_{i0} plane (the third and fourth directions). While this has the same multivector form as the electric field, and its geometry is fixed by the actor frame of the electric and the magnetic field, it is observed in the observer frame in

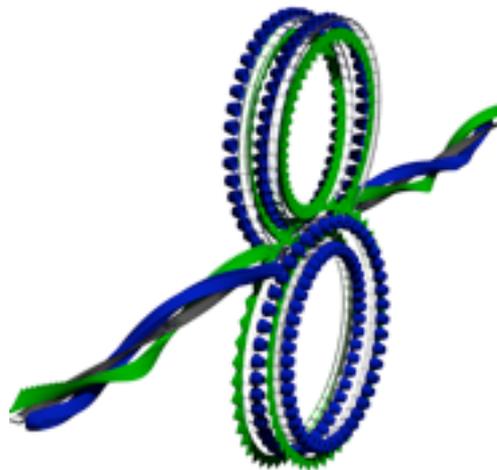


FIG. 1: Diagrammatic representation of the electron positron pair creation process at threshold, showing field configurations before and after the interaction. A single wavelength λ_c of each incoming photon is shown. The size of the vortices have been emphasised. The loop radius is $\lambda_c/4\pi$ and would otherwise be difficult to distinguish on the scale of the drawing. The electric field vector direction is represented using blades (green), and the magnetic field direction using truncated hollow cones (blue).

which it, indeed, performs an oscillation. Which momentum is measured will depend on the direction in which it is measured and will have an eigenvalue corresponding to either plus or minus E/c , depending on the phase at the point of measurement.

Configurations with radial magnetic field, though they may exist briefly in high-energy interactions, are not confined since the magnetic fields terms cancel identically in the product. That is free magnetic monopoles cannot be constructed within the present framework. It is entirely possible, however, that such short-lived states may produce interactions, and it is speculated that these may be identified with the weak interactions.

In the pair annihilation process the initial fields in the vortices will be destroyed only up to a rotation horizon imposed by the speed of light and the rotation (Compton) frequency of the initial particles. This leaves an ephemeral hole in the field, with the opposite topology of the particles formed. This hole, being electromagnetic, would clearly propagate at lightspeed. It is speculated that such holes may constitute neutrinos.

The picture presented above brings with it its own set of conceptual problems. In which space, or in which combination of spaces does the vortex exist? It will clearly not be the normal 4-vector space since its characteristic components are such things as the electric field, the magnetic field and the electromagnetic momentum density, each of which, as discussed above, is described by its own 3-component (bivector) space. Lightspeed motions contract space and time and modify the observed

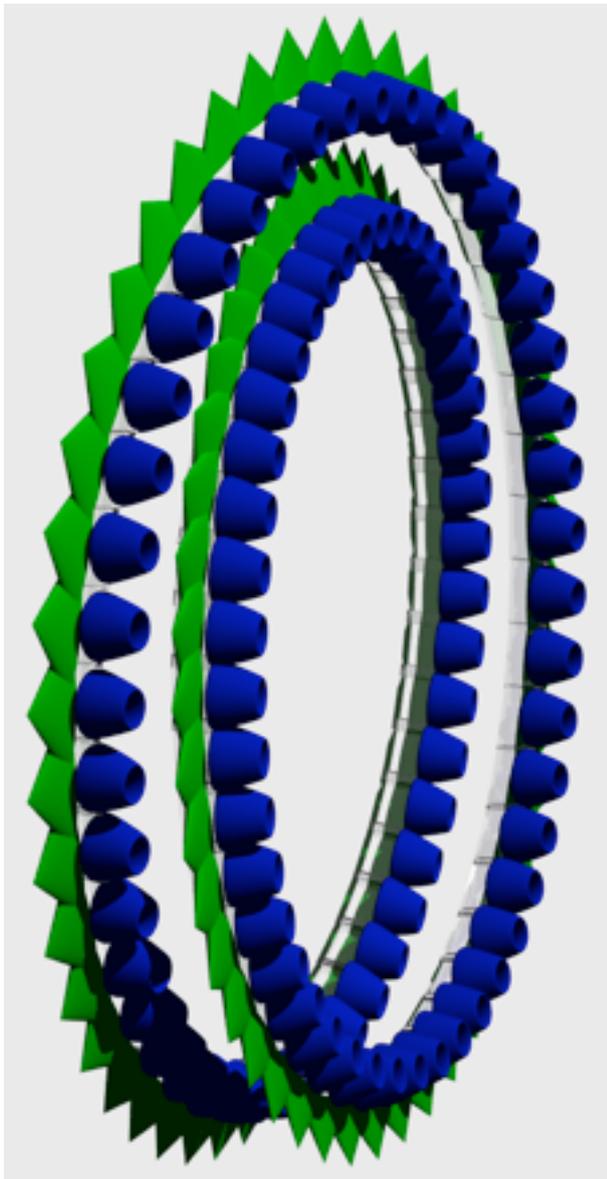


FIG. 2: Field snapshot for the positron at creation. The plot represents a nine-dimensional distribution, in which four (bivector) directions are important. The electric field vector direction is represented using blades (green), and the magnetic field direction using truncated hollow cones (blue). The glass lozenges represent one path of field momentum flow. The full structure fills space with tumbling toroidal shells, and is spherically symmetric[1]

fields, so what is an appropriate frame for an element of the object? If it has a characteristic frequency (for the electron the Compton frequency is just equal to the incident photon wavelength at the pair creation threshold) and the motion is rotational, then the finite speed of light imposes a rotation horizon[22] on the internal flow of energy-momentum. How can one reconcile a rotational

motion of the order of the Compton wavelength with the observed structureless, apparently spherically symmetric electron?

Several of these issues were addressed in earlier work on a simple semi-classical model of the electron as a confined photon[1]. In that work the confinement forces were not considered at all but simply postulated. This corresponded to replacing the postulated Poincaré stresses which bind the electron charge, with a postulated (self) confinement mechanism for a photon at sufficiently high energy. Given a starting point of such a confined photon, it proves relatively straightforward to calculate the charge, the spin and its apparent size in high energy interactions. The charge of such a state was found to be close to the elementary charge, the spin to be (precisely) half integral and the apparent size found to scale with the scattering energy. The calculated charge was length-scale independent and depended only on the topology of the photon vortex. A possible origin for the exclusion principle and a limit on its realm of validity was proposed. The origin of de Broglie's Harmony of Phases was proposed and hence the de Broglie wavelength of the object derived. As is well-known, such a starting point is sufficient for the development of a quantum mechanics.

By considering the horizon imposed on a rotating electromagnetic object by the speed of light, we were able to propose a physical origin for the anomalous magnetic moment of the electron and found a value for this anomalous moment in agreement with first order Quantum Electrodynamics.

Eq. (23) requires that it is the Electric field which is radial and the Magnetic field axial. The combination of rotation and twist result in the electric field executing a double loop and the magnetic field direction remaining fixed in an axial direction, at least on the eye of the torus. That is, this field distribution is an electric monopole and a magnetic dipole configuration. That the objects are fermions, though originating from bosons, is evidenced in (at least) three ways, presented in increasing order of importance. Firstly, it is a simple matter to calculate the momentum and that is numerically half-integral[1]. One may then invoke the spin-statistics theorem and argue that it must be a fermion. Secondly, the electric field returns to its original configuration after a 720 degree rotation and this results in an essentially fermionic field distribution. After a 360 degrees one is on the other side of the torus (with an opposite sign in one axis of toroidal co-ordinates). Thirdly, and most fundamentally, the field configuration exhibits that defining trait of fermions: trying to put two of them into the same state results in enormous (stronger than the "strong" force) resistance. The reason for this is that the fields add linearly whereas the energies add quadratically, requiring an energy input of the order of the masses of the particles involved. Only for the precisely spin antiparallel case is there no energy cost, since one of the fields (magnetic) cancels. That the force is far stronger than the supposed "strong" force has been amply demonstrated by experiments in the seven-

ties, and confirmed many times since. Note that such experiments also cast doubt on the identification of quarks with partons[23].

In forming the figure we have assumed effectively that all of the energy-momentum of the initial photon was defined at a single point with a single electric and magnetic field- a clearly unphysical assumption and an extreme limit. Were this the case, however, the torus illustrated would undergo a rapid tumbling due to conservation of momentum, resulting in a time-averaged spherically symmetric distribution in momentum space. In reality, however, it is more likely that both the initial field is spread over all phase angles, resulting in a spherically symmetric distribution and also that the momentum flow forms a mode structure, under the constraints of the rotation horizon, giving a set of non-crossing paths filling all of the (momentum) phase space available to it. For a picture of how such a structure would appear in momentum space the reader is referred to earlier work[1]. In the absence of an external magnetic field, such a structure would have a radial Electric field and a zero (at least on average) Magnetic field. The presence of an external magnetic field, however, would break this symmetry resulting in an electric monopole and a magnetic dipole field. A very extreme magnetic field (of the order of that present in pair-creation) would force the electron into the configuration illustrated.

The total mass of any (self) confined structure originating from an incident photon will be $m = U/c^2$, where $U = hc/\lambda$ is the energy of the constitutive photon of wavelength λ . It is clear that for the case where the electron and positron annihilate at rest, the resulting photon wavelength λ is just the electron Compton wavelength $\lambda_C \equiv h/m_e c \approx 2.43 \times 10^{-12}$ m. In the case where both loops lie exactly on top of one another the loop radius is $\lambda_C/4\pi$ and this is the characteristic length scale of the model. The circulation repeats itself with period half a wavelength. In flat space, this would lead to total destructive interference everywhere along the path. Within the twisted loop the interference is always constructive as is clear from the figure, provided that the period of rotation coincides with the phase rotation of the fields. That is, for a solution, there must be a harmonic relation between them. This harmonic periodicity leads to a quantisation of the continuous fields, where the oscillation is carried, just as in the simpler case of the Maxwell equations, through the fields. In other words what is waving is the fields and what it is waving in is the pivot provided by the invariant scalar.

There are (at least) four rotations in four distinct planes about three distinct axes but with a specific phase relation between them. These are: one rotation of a vector along the eye of the torus about the origin, a second one consisting of a twist of the momentum about an axis along the eye of the torus, a third one consisting of the rotation of the electric field vector about the path element defined by the momentum flow and similarly a fourth one of the magnetic field vector about this same axis. These

are fixed in the phase ratio of 2:1:1:1 for the object illustrated in Fig. 2. Note that these rotations are not only in different hyperplanes but also do not all share the same centre of rotation. The resultant combined motion is quite complicated but because of the harmonic phase relation between the different rotations, not completely unimaginable. We call this motion the quantum bicycle, because of its partial resemblance to the motion of two massive spinning wheels, with fixed relative phase (cogs and gears) rotating about two perpendicular axes in free space which do not share a common origin. The actual motion is, of course, a little more complicated than this due to there being at least three axes (on the assumption that the electric and magnetic field share a common axis) with rotations in different linearly independent hyperplanes. There are many illustrations of such motions on the internet. See, for example, Fergus Murray's excellent applet which contains projections of three simultaneous rotations, and looks (of course) both toroidal and spherical. The characteristic frequency of the object is twice that of the photons which formed it. The di-photon state has twice the energy of the initial photons and hence should have twice the frequency. In the final state the doubled frequency is evident in that the configuration executes a double loop before returning to the starting configuration.

The present algebra allows a much richer description of solutions than is possible with a complex algebra. An exponential of any of the ten unit elements which square to -1 , for example, represent a "vector" in that space formed by the scalar and the unit element in question. By analogy with complex exponentials, such forms may be used to describe oscillations. Pre or post multiplying by another unit element converts this to a rotation in a space consisting of any desired pair of unit elements. The ordering matters in a non-commutative algebra, and may result in a change of the direction of rotation, or of the handedness of elements with respect to each other. For example, if we choose a Cartesian frame for the representation and assign the unit vectors γ_i to the normal Cartesian vectors, $\gamma_1 = x, \gamma_2 = y, \gamma_3 = z$, we may then write a vector in the $x - y$ plane with angle θ to the $x - axis$ as $\gamma_1 e^{\gamma_{12}\theta}$, where the sense of rotation is right-handed. Such combinations parallel the properties of complex exponentials in simple forms, but may have other desirable properties in their more complex manifestations. For example, unit elements which square to $+1$ (of which there are six) describe, by themselves, falling or rising exponentials. This means that introducing a 3-vector or a 4-vector into the exponent can describe an object propagating in one dimension, but exponentially confined laterally. This is an option not open to a merely complex representation. For example, amongst other possibilities, the incident photons may be represented by [6, 16]:

$$F = F_0 e^{\gamma_{0jk}(\gamma_i k x_i - \gamma_0 \omega t)} = \gamma_{0j} (1 + \gamma_{0i}) e^{\gamma_{0jk}(\gamma_i k x_i - \gamma_0 \omega t)} \quad (24)$$

The expression has been written in such a way to make

explicit the proper form of space and time in the exponent. As one propagates through space and time the field pattern is conventional in that the electric and magnetic fields corkscrew around the propagation direction, but unconventional in that the fields are, like a photon, strongly confined laterally. For two such objects describing two photons of the same helicity in a counter-propagating configuration such as that described above for pair-creation, the exponential parts cancel leading to a stationary pure-field configuration (corresponding to the twisted-mode discussed above). Clearly, such a state will not stay stationary for long. One possibility is that it reverts to two photons, and this will always be the case below the pair production threshold. Above this however, where the pivot term may be strong enough to provide a local rotation, one may write down a rotator (as opposed to a propagator) solution. For example, a radial Electric field in the $\gamma_{10} - \gamma_{20}$ plane may be written: $E_0\gamma_{10}e^{\gamma_{12}\theta}$. This, by itself, corresponds to the field configuration illustrated in the figures, with E radial (and planar) and B axial. Using the same rotator, the magnetic and pivot components may be written in (at least) two ways, differing from each other by a phase shift of 90 degrees. $B_0\gamma_{12}e^{\gamma_{12}\theta}$ or $P_0e^{\gamma_{12}\theta}$. Combining the two, that is writing, for example $(P_0 + E_0\gamma_{10})e^{\gamma_{12}\theta}$, leads to a periodic phase-locked reversal of the (actor) B field, and hence the whole object tumbles, leading to an (on average) three-dimensionally radial Electric and an (on average) zero Magnetic field. This corresponds to an electron solution.

Note that this solution is very similar to that proposed by Dirac for the electron[9] (p263), but expressed as Fields using the properties of the Clifford-Dirac algebra. He showed that one can analyse the electron motion into a part that describes the (relativistic) motion of the electron as a whole plus a light-speed oscillatory part of twice the Compton frequency $2\omega_C$, the so-called “Zitterbewegung”[24]. Further, any instantaneous measurement of any component of the electron velocity will *always* yield one of the eigenvalues of $\pm c$. This “Zitterbewegung”, the frequency $2\omega_C$, and the instantaneous velocity eigenvalues of $\pm c$ are clearly also features of our solution. The solutions are not the same partly because the Dirac algebra used by Dirac himself was different to that used here in that it was not also a simple Clifford algebra, and partly because he did not start from the fields. Properly his algebra was the product of the algebra used here with the Complex algebra. This additional complexity may have obscured the nature of some of the extra terms. In Dirac’s words (p265). “These extra terms involve some new physical effects, but since they are not real they do not lend themselves very directly to physical interpretation”. The development is famous for revealing the nature of fermionic spin. In retrospect, the “new physical effects” refer to the nature of charge. This could not be inferred from the Dirac approach, however, primarily because he did not start from the electromagnetic fields.

Along a different line, an interesting development of the Dirac model, though again not using the fields, was made in the geometrical “Zitterbewegung interpretation of Quantum Mechanics” developed by Hestenes[25]. In this work it was shown that the Zitterbewegung may be used to interpret the half-integral electron spin and that the trajectory of a moving Dirac electron may be viewed as series of light-like helices of radius $\lambda_C/4\pi$ defined by the rotation of the electron energy-momentum flux in a plane perpendicular to the spin[26, 27]. This is just the trajectory of the eye of the torus for our model (Fig. 2).

Let us now discuss the stability of the object proposed. The object is both twisted and looped and once created both of these must be removed to undo it. Clearly, in the absence of the proper antiparticle, the complexity and extent of the vortex and its external field will require some undoing. Even vortices in simple fluids maintain their integrity over many revolutions. A serious initial barrier to dissolution is the conservation of angular momentum; to this must be added the removal of a non-trivial field topology, and the annihilation of quantised charge and spin, and some mechanism to undo the now fermionic double-looped nature of the object. Note also that the distribution illustrated is a minimum energy configuration. The dual form in which the Magnetic field is radial and the Electric axial, a magnetic monopole, would have much higher energy due to the much stronger coupling constant for the Magnetic field[5]. Such a particle would have much higher mass than the electron and be extremely unstable against decay into an electron, since all that is required is a 90-degree rotation of the constituent photon axis. Also, as argued above, it is simply not bound.

This does not preclude such objects being formed in short time scales in high-energy collisions and, as noted above, it is tempting to speculate that the dual electromagnetism so formed may be responsible for the weak force. Note that this means that any slight deviation from a perfect radial electric field in this direction will also incur a huge energy cost, making the field configuration illustrated extremely stable. Energy may be added to the object, leading to an increase in the magnitude of P and a slight tightening of the loop, or may be emitted by the object, leading to a slight loosening of the loop, but the essential form of the twisted-looped electromagnetic object will be maintained. The intrinsic properties of the object, its charge, spin and flux quantum number are unchanged in a change of scale[1]. In summary, stability is ensured by energy, angular momentum, topology, fermion, charge and spin conservation.

In the pivot version of the Maxwell equations the observed charge is related to the time rate of change of the pivot. This would, in the absence of interactions, require a monotonically increasing rest mass for particles through the P^2 term in the extended Maxwell equations. The sign of the pivot, however, is opposite for opposite charges, leading to an energy reduction for both on mutual exchange, and hence an attractive force between

them. Conversely, a like charge in the vicinity would increase the local magnitude of the pivot, leading to a repulsive force between them. The likely environment for the median electron is in the locality of a proton, bound in a Hydrogen atom. In this situation there is a continuous exchange between the two of square-root energy (P) each providing the opposite sign to hold the mass of the other in check. Clearly, the interaction is not merely local in space, but local in space-time.

Each twisted-looped object will attempt to reach a local balance through its interactions with all other particles in its universe. This universe consists of all points on the lightcone centred on the particle, and hence spans the spatial universe, albeit at different times. The level of the pivot required to maintain the size of the loop relevant for local conditions is automatically adjusted at very high frequency (order of the fine structure constant times the Compton frequency) by the many (electromagnetic) interactions with all other particles in the universe. The magnitude of the charge is related to the fact that it is an interacting looped, harmonic, resonant quantised object which has a certain probability of exchange with some other particle somewhere in the universe. Conversely, the sum total of interactions with the rest of the universe will produce a reaction at the surface of the particle, an essentially radially inward directed force, which may be another of the Poincaré stresses and should then be added to the force equations mentioned above. The electromagnetic equations can accommodate any radial interaction level simply by a marginal change in the level of the pivot.

Note that such a conceptual approach leads naturally to a physical basis for Mach's principle. Note also that the charge, being related to the likelihood of (photon) exchange per cycle, may also change gradually with epoch, or have slightly higher values in regions of high local density. Some of these effects may be accessible to experiment. It will not have escaped the reader familiar with Quantum ElectroDynamics (QED) that the framework sketched above incorporates its essential features, except that the source is now itself a purely electromagnetic vortex with a limited length scale $\lambda_c/4\pi$. It is possible that this length/energy cutoff may remove the need for renormalisation.

VI. OTHER PARTICLES: COMPLEXITY FROM SIMPLICITY

Earlier we argued that experimental fact that the electron positron pair could be created from pure electromagnetism, or annihilated to pure electromagnetism should mean that both electrons and photons should be described as purely electromagnetic phenomena. Clearly, the same argument is true for most other particles, since most may be created as particle/antiparticle pairs. A simple extension of the ideas presented above allows one to propose simple electromagnetic building-blocks which

may be used to construct most of the plethora of particles observed in high-energy scattering experiments. As will rapidly become clear, the model, if it were to be correct, would also afford a unification of the strong and electromagnetic interactions, at least if the removal of the necessity for an exchange particle in the form of a gluon constitutes a unification.

It has been argued above that the electron may be described by a single wavelength complete electromagnetic loop. A simple extension to this is to consider that the loop may loop again. In Fig. 2, one sees two strands of momentum flow because of the double-looped configuration. For an isolated particle, because there is a toroidal mode structure and the torus tumbles, one sees two strands in any direction. Because of the freedom afforded by the extent of the space, one may imagine looping this again. One would then see four strands for one complete circuit. It is tempting to ascribe this doubly looped object to the next generation lepton the muon. It is amusing to play a numerical game with such an object. The combinatoric number of double looped (electron like) objects in a four looped (muon-like) object is 216. This comes about because there are six sets of pairs which can be made from four objects, times the three (bivector) directions in which the complete object is bound gives $6^3 = 216$ possible "box pairs". This number is fairly close to the mass ratio of the muon and electron. One may play the game again. Looping again gives six strands per dimension. This, then, would correspond to the tauon generation of lepton. The number of pairs which can be formed from 6 is 15. $15^3 = 3375$. This is very close to the mass ratio for the electron and tauon. This may be a co-incidence, there are only two ratios and other numerological co-incidences may be more convincing. There is also, of course, no suggestion that there are somehow 3375 electrons in a tauon. A proper confirmation of whether this had any substance would need a consideration of the mass ratios of other particles as well. It may, however, be fun to look for the fourth generation lepton at around mass of $28^3 = 21952$ times the electron mass in the new collider at CERN, though this may have too short a lifetime to be observed.

What about the nature of hadronic particles? In the standard model these are described as quark-antiquark pairs (mesons) or three quark, or three antiquark sets (baryons). Quarks have never been observed as isolated particles, and even the charges ascribed to them in the standard way are hard to measure[4]. They are supposedly bound by a strong interaction intermediated by an exchange particle, the gluon, which also couples to itself. This self-coupling is supposed to explain why free quarks have not been observed (trying to do so generates a lot of glue). Such a paradigm, however, raises its own problems such as why glue only states (glueballs) have not been observed and why only particles corresponding to three quark and quark-antiquark pairs are observed in nature. What is wrong with 5 or 42 quark objects for that matter? Further, as mentioned above, there is outstand-

ing experimental evidence from polarised proton scattering that the quark-parton model (which identifies quarks with the partons responsible for the scaling (point-like) behaviour observed in deep inelastic lepton scattering experiments) is invalid[23]. What is needed is a model for the quark symmetry that does not involve the quark as a separate particle. In the present context, a remarkably simple conjecture can provide such a symmetry, and perhaps clear up a lot of the gum in our understanding of hadrons.

The main thesis of this paper is that the paradigm for particles is that they should all consist of complete, resonant, harmonic, electromagnetic loops (or loops of loops etc). Can we design an electromagnetic building block which, although not a complete path in itself, can be combined with other such objects in order to form complete looped objects? The answer is yes, and it is remarkably simple. It lies in considering which classes of partial electromagnetic loops may be combined to create complete objects. We will consider configurations whose end result is a complete change of direction (e.g. x to y and so on). Consider firstly an object, a region of looped field, whose end result is a quarter turn in the direction of field momentum, this may be combined with another such object to create a half turn, or a different quarter turn. Four in the same sense make a complete turn, but this is just an electron or other lepton. The interesting case is of objects whose end result is to generate a change of direction at, or near the origin. One such object is the five-quarter turn, a complete loop, but an overshoot.

Consider a region of invariant scalar pivot P strong enough to form a complete looped object such as an electron. In particular, let us imagine that it executes a loop such that the incoming photon direction is transformed through 90 degrees (say from x to y). Such an object is not a complete path in itself, and neither is a second such object following on from it. There is, however, a configuration in which three such objects may form a complete object that is, for example x to y y to z and z back to x. That is three loops in the same sense (that is with the same sign of P) may be combined to form a complete object. It is this sort of oriented loop which is identified as a “quark”. Any such loop (for example a double loop with an overshoot, corresponding perhaps to a strange quark) could be bolted together in sets of three (in a trefoil configuration) to form particles. As is well known, such a symmetry generates the observed spectrum of baryons. Another possibility to form a particle is to combine a loop in one sense (x to y) with a reverse loop in the opposite sense (y to x)(identified with an antiquark). This means that loop-antiloop (quark-antiquark) pairs would also form particles, in a figure of eight configuration in the bivector space. Again, it is well known that such a condition generates the observed hadronic mesons.

Despite the similarities in the predicted particle spectrum, there are major differences between this view and the standard model. Firstly, the reason for three quarks and quark-antiquark is geometrically self-evident. Fur-

ther, in the standard model a new exchange force, the strong force, is proposed. Here there is no need to introduce a further force to bind the quarks, as this is already present through the action of the pivot. The (SU(3)) properties of the exchange particle are replaced by a condition of continuity (that the resultant path is closed). One is used to thinking of electromagnetism as a relatively weak force, but this pertains to exchange (of photons) as in QED. The force represented by the time derivative of the $P\vec{E}$ is as strong as you care to make the pivot and the field. The changing magnetic field is, as mentioned above, hand-of-god-strong in the Maxwell equations. In the Maxwell equations it is an irresistible condition with undefined strength. The intimately electromagnetically bound electrons in the metal of a spark plug, if they could talk as they were ripped out and forced to jump a millimetric gap through an effective vacuum, might disagree that changing fields themselves constituted a weak force. Note also that even the derived force of the attempt to overcome the uncertainty principle is, experimentally, already far stronger than the “strong” force of the standard model[23].

VII. DISCUSSION

To what extent is the model consistent with relativity, QED, Maxwell and Dirac quantum mechanics? Relativity is built in with the maths. It has been argued that the model provides a new basis for QED, and reduces to Maxwell’s equations for zero invariant scalar pivot. So yes for the first three. The fourth merits some further discussion. As mentioned above, there is a sense in which the full derivative of the electromagnetic energy momentum $d\frac{1}{2}(F+P)(F+P)^\dagger$ is similar in structure to the Dirac equation. Both have the same derivative (d is identical to the Dirac derivative) but the Dirac approach operates on a wavefunction defined in a spinor space and adds a mass term by hand, whereas the present approach acts on a field-pivot product in which mass terms arise naturally. It is also worth noting that the spinor space of Dirac is a subset of the Dirac-Clifford algebra used here. One difference is that Dirac solutions are often expressed using a unit imaginary, which is not present in the present algebra, but this may not be necessary[25]. The Equation has all the beauty and simplicity of the Dirac equation but enhanced by the beauty of the Maxwell fields.

We now discuss whether or not the present model answers the question of the electrons enigmatic size. Point-like and smaller than attometres in deep inelastic lepton scattering[4], classically bigger than the classical radius of 2.8 femtometres with a Compton wavelength of 2.4 picometres, looking at least hundreds of nanometres in the solid state[7], and in superconducting systems being macroscopic[28]. The answer is a qualified yes but only to an extent. In the above, the only clear contradiction is between the effective size the electron is observed to have in deep inelastic scattering, and the size limit im-

posed classically by the electron mass. That contradiction can be resolved. The present model views a lepton as a confined rotating electromagnetic vortex. Any interactions between such objects will be via a virtual photon exchange. Such an exchange photon always has greater wavelength than the characteristic lepton size, as it is minimally $\lambda/2$, whereas the converging parts of the rotating vortices scale relativistically in the same way as the exchange photon and hence are always, at any energy, at least a factor of 2π smaller than this[1]. That is the apparent lepton size shrinks at precisely the same rate as does the size of the resolving photon. Hence the interaction appears pointlike at any energy.

This point has caused much confusion in the past, most of which has stemmed from a lack of understanding of the difference between a point particle and a point-like particle amongst those not familiar with a field such as deep inelastic lepton scattering. The electron is not a point particle. A point particle cannot have spin, cannot exhibit a wavelength and, with charge, would have an infinite mass. The electron is experimentally, however, a point-like particle under lepton scattering up to the highest energy electromagnetic exchanges achieved so far and that is quite a different thing. For an object to have a point-like interaction it is necessary that it be a single object (in contrast to a composite object such as a proton). Point-like implies that an exchange photon in the scattering process between two electrons resolves neither of them. That is the case here since, at high energy, the apparent size of the confined photon scales relativistically in exactly the same way as does the exchange photon. For an interaction to be point-like one requires a single object of an extent which is not resolved by the scattering process. This is clearly the case here and hence the object can both have a finite size and a characteristic length scale (though this size scales relativistically) and, at the same time, have a point-like interaction in high energy experiments.

As to the other side of the question, how the electron can appear so huge in the solid state or in superconductors. No, at least not yet. At least the fact that all particles and their interactions are purely electromagnetic, together with the fact that the electron can lose energy through size relaxation, gives hope that such a possibility may be describable within the present framework in the future. Such a model would require a redevelopment of the work on collective phenomena, which, while perfectly feasible, is beyond the scope of the present work. The interested reader is referred to a recent beautiful monograph on Collective Electromagnetism by Carver Mead[28] which goes a long way to describing such phenomena based on the paradigm of the phase change induced by the flux quantum. Both the flux quantum and the second order equation he uses to replace the Maxwell equation, are derivable from our first-order approach, so a combination of these two approaches may prove fruitful.

In any quantum formalism, the electron has some sort

of field and some sort of phase. In fact, as is well known, there must be two oscillations associated with the quantum electron[29]. The oscillations transform differently under a Lorentz boost but share as single phase defined at each point in space and time. One oscillation slows down under a boost obeying the relativistic law of the slowing of clocks, while the other speeds up obeying the relativistic law of increase of frequency with energy. This occurs in such a way that both oscillations remain in phase with each other, for all space and for all time. This is de Broglie' law of the Harmony of Phases. It was a proper consideration of the relativistic nature of clocks under a boost (slow down) and frequencies under a boost (speed up) which led him to propose his famous relation $\lambda = h/p$ and in turn led to the development of Schrodinger quantum mechanics. Because of the difficulty of the underlying concept, a discussion of this process is, sadly, now absent in most graduate and undergraduate texts. Indeed, the author has come across papers in the quality press, where this property has been published as though it is something new. It should be clear that the various harmonic components of the model discussed above, the fields, currents and rotating momenta, will transform differently under a Lorentz transformation. Some apparently slowing down (for example the rotation), others apparently speeding up (for example the rate of phase change of the field components). Since all originate from the same process in the rest frame of the electron, where they clearly share a common phase, this common phase will be reflected in the frame of any external observer as well.

It should be clear that what would be observed of the object illustrated in Fig. 2 depends, as is usual in a quantum state, on exactly how it is observed. The figure as drawn corresponds most closely to fixing the direction of one of the field directions, for example that of \vec{B} in a (very) strong external magnetic field. Consider, firstly, all points of equal phase. Close to the eye of the torus this corresponds to a slice through the mode structure. This resembles a ring close to the eye of the torus but becomes progressively more distorted as one moves out, like the petals of a spherical flower. This distortion takes two forms. The first is that familiar from toroidal coordinates, with the point at the origin corresponding to that at infinity under a phase change of the twist of $\pi/2$. The second arises from the twist of the momentum flow around the toroidal surfaces and means that the phase front becomes more and more twisted as one moves out, remaining perpendicular to the direction of momentum flow, in the sense of the projection rules. The shape of this phase surface is not, of course, directly observable and must be inferred by analogy with constrained fluid flow. The direction of maximum rate of change of phase corresponds with the rotation of the torus only at its eye, because of the twist of the path about this direction at the eye of the torus the fields in both loops are everywhere parallel and everywhere interfere constructively. It is at the eye of the torus that the harmony of phases

discussed above is defined.

To gain yet another perspective, it is instructive to consider the path that the electric field vector would follow if we were to hold the magnetic field vector fixed in a certain direction in space. For the eye of the torus the electric field vector would sweep out the path of a lighthouse beam, in an equatorial plane. As one moves out from the eye, the path would begin to resemble that of a vector drawn from the centre of a tennis ball to its seam. At the limit one has a beam in the polar directions. All paths on the surface of a sphere are swept out. That is, for the proper distribution of energy on the toroidal shells, this constructs a spherically symmetric electric field distribution in the basis of $(\gamma_{01}, \gamma_{02}, \gamma_{03})$. In this respect the torus corresponds to one stereographic projection of a hyperspherical surface in three dimensions of space and one of time, whereas simply projecting out time, as in the conventional projection of the electric field, leads to a simple spherical projection.

Note also that, because the electric and magnetic field vectors live in linearly independent spaces, it is quite possible to define each of them on separate toroidal surfaces, which need not necessarily be on top of one another. In particular, it is even possible that the electric and magnetic field components may propagate in opposite senses through the ring, provided only that the sense of the twists were also reversed so that the phase relationships were maintained. It is worth noting that the solutions proposed have this property.

Finally, we have argued that the object has half integral spin. Can we throw any light on that most peculiar property of quantum spin, that it always appears parallel or antiparallel to the axis of the measuring device? The answer is yes. To measure the angular momentum of the object, one must perform a projection of the internal fields, defined in γ_{i0} space, onto some measurement axis (γ_j). This measurement axis is defined, not by the object(actor), but by the measuring device(observer). A nice analogy is that of shoving a stick into the wheel of the quantum bicycle. Only a perpendicular stick will do, otherwise an angular momentum object will not be formed but something which is time-like. It has been argued that the electron will hunt through all the bivector phase space available to it within a cycle. Within a fraction of an attosecond this condition will be met and all of the angular momentum will be resolved, either parallel or antiparallel onto the measurement device. The author must admit to particular personal pleasure in this one, as this curious property of quantum spin was one which caused him many moments of puzzled incomprehension over many years.

VIII. EXPERIMENTAL TESTS OF THE MODEL

The most convenient avenue for the experimental testing of the model proposed would be a study of the sub-threshold precursor state of electron-positron pair cre-

ation. Such a study would require two gamma ray lasers each operating at about 500MeV. In the absence of such a facility, it may be possible to observe some effects at much lower energy. Looking carefully at the twisted-mode configuration, the low-energy precursor of pair creation, one may observe photon-photon interaction and, possibly, radiation. In a twisted mode two laser beams of the same circular polarisation are incident on each other from opposite directions. This gives rise to a field structure where \vec{E} is parallel to \vec{B} everywhere (and for all time). The Poynting vector of the combination is therefore zero and in this sense we have to do with no electromagnetic momentum flow and hence, in a sense, this is already stopped light, though with insufficient energy to form a pair. Since stopped light has (rest) mass, it may be able to radiate. Such a process may provide a component of the observed red-shift of light from far galaxies. If so, this may provide an alternative origin to this radiation to that of a fossil big-bang.

IX. CONCLUSIONS

It has been argued that physics is best described with reference to that algebra, whatever it is, that best parallels the way the universe itself is put together. It has been shown that using the best current candidate for this, a Dirac-Clifford algebra, leads to a natural basis for a formulation of Maxwell electromagnetism. All the Maxwell equations may be expressed as $dF = J$ with the correct signs and with nothing more or less. In particular, the formalism does not require a field and a dual field to represent the inhomogeneous and the homogeneous equations respectively.

Sixteen abstract linearly independent basis elements are introduced through multiplication (or division). These are not obscure, but represent such things as unit rotations or boosts. In this framework the proper unit basis of familiar objects have been discussed and it has been argued that these all have the proper relativistic transformation because they are formulated from the beginning in a proper basis. The geometrical relationships amongst each other of such things as the field, current, angular momentum, time and invariant scalar quantities of electromagnetism have been considered. We have made explicit the projection rules which have been used historically to describe the parallelism or perpendicularity of the various linearly independent basis elements with respect to each other. All transform in the same way under translation and hence the various electromagnetic quantities have a simple relation with respect to one another in the case of a plane electromagnetic wave.

Because the quantities do not transform the same way under such things as rotations (or rotations of rotations) and boosts, however, the situation is more complicated for an electromagnetic vortex such as that discussed in earlier work[1]. There, a simple model of a self-confined single wavelength photon was presented and we

were able to show that the resulting field configuration would have charge, spin and anomalous magnetic moment close to that of the electron and positron, as well as a point-like interaction in high-energy electron scattering experiments. It was argued that the object was a fermion because it had half-integral spin, a factor of -1 on 360 degree rotation and, most fundamentally, allowed a derivation of the exclusion principle. The present paper has clarified in which space such a vortex may be formed. This is the space of (bivector) momentum rather than the spatial part of the four-vector.

For the present work we have investigated how such an object might be bound by a simple extension of the Maxwell equations. It is found that the key to understanding how a propagating electromagnetic field might be curved into a vortex lies in a proper consideration of an invariant scalar term. Because of its properties in generating rotations and confinement in this formalism, we have referred to this term as the pivot. A consistent solution for a simplified equation has a radial electric and an axial magnetic field in the rest frame. Electric monopole-like vortices are bound, and magnetic monopole-like vortices are not.

A purely electromagnetic model for “quarks” has been sketched. This model gives the symmetry of the quark model, without the need to ascribe the quarks to real particles. Instead, they arise as a property of continuity in three-dimensional (bivector) space. Since there are no particles, only fields, there is no need for an extra exchange force to bind the particles together. Instead, it has been argued that there are other strong forces in the extended electromagnetism which are responsible for the binding of leptons as well as hadrons, and these may be described by the generalised Lorentz forces presented,

or through interactions with the rest of the universe. We have suggested a possible route for the experimental testing of the model, though others on the same lines may be devised.

Finally, whether or not the present formalism proves to be correct in detail, it is hoped that one thing is clear. It is vital to find a theory which allows a continuous description of the transformation of light to particles and vice-versa, as the experimental evidence that this is essentially the case is overwhelming.

Acknowledgments

The work presented above is derived from a long standing collaboration with my friend and co-pirate (eyepatch on the other eye) Dr. Martin van der Mark. At least half of the credit for anything creditworthy in the above is due to him. Any mistakes or omissions, however, are the sole responsibility of the author. Many thanks to Stephen Leary for not jumping off the cliff, but rather helping to scale it. It is a pleasure to acknowledge Phil Butler who came from the other side of the planet to introduce us to the algebra. Thanks to Ariane Mandray for being the first to visualise the matching between the vortex fields and the photon fields. Thanks to Prof. John Barker for being the first to clearly articulate himself over his confusion of the form of the vortex solution in the original paper. Thanks to Grahame W. Williamson for proposing the term “rotation horizon”. Thanks to Tim Drysdale, John Weaver, Prof. Michael Berry, Nick Green, Tony Booth, Ulrich Enz, Phil Burt and Prof. H.B.G. Casimir for valuable conversations.

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