

# Is the electron a photon with toroidal topology?

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## Abstract

We study the properties of a simple semi-classical model of a photon confined in periodic boundary conditions of one wavelength. The topology of this object, together with the photon electric field, give rise to a charge of the order of  $10^{-19}$  Coulomb and a half-integral spin, independent of its size. The ratio of the electromagnetic energy inside and outside the object leads to an anomalous spin  $g$  factor which is close to that of the electron. Although a finite size of order  $10^{-12}$  meter arises in a natural way, the apparent size of the object will be much smaller in energetic scattering events.

Nous étudions les propriétés d'un modèle semi-classique simple d'un foton renfermé dans un domaine d'une seule longueur d'onde. La topologie de cet objet, avec le champ électrique du foton, amènent à une charge de  $10^{-19}$  Coulomb et à un spin demi-intégrale, qui sont indépendents de sa dimension. La proportion de l'énergie électromagnétique dans et hors de l'objet donne un ratio gyromagnétique très proche de celui de l'électron. Bien que l'objet à une dimension de  $10^{-12}$  mètre, la dimension qui pourra se manifester dans les expériences à haute énergie sera beaucoup plus petite.

# 1 Introduction

During the past century there has been much interest in describing elementary particles purely as field phenomena. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] The generation of charged particles from uncharged fields,[18, 19, 20] and the creation of fermions from bosons[21, 22, 23, 24] have been addressed in the literature. Despite this work, the physical origin of charge and half-integral spin are not yet fully understood.

Within the current theory of charge-photon interactions, quantum electrodynamics, both the charge and mass must be left as free parameters, the charge and mass of the bare electron diverge, and a renormalisation scheme must be adopted to impose the physically observed values  $e$  and  $m_e$ . This implies that quantum electrodynamics, as it stands, cannot describe the origin of charge and mass, as these degrees of freedom have already been used in setting up the theory.[25, 26] Despite its tremendous success in describing the interaction between electrons and photons, the theory does not address the internal structure of the electron.

In the classical Abraham-Lorentz theory of the electron,[26, 27, 28] the energy contained in the Coulomb field of a charge  $e$  in all space outside its radius  $R$  is

$$U_{elec} = \int_{|\vec{r}| \geq R} \frac{\epsilon_0}{2} E^2 d\vec{r} = \int_R^\infty \frac{e^2}{8\pi\epsilon_0 r^2} dr = \frac{e^2}{8\pi\epsilon_0 R}. \quad (1)$$

For a point charge, with  $R = 0$ , the total energy  $U_{elec}$  is infinite. The physical mass of the electron,  $m_e = U/c^2 = 0.511 \text{ MeV}/c^2$ , then imposes a lower limit on its size of the order of the so-called classical electron radius  $r_0 \equiv 2R = 2.82 \times 10^{-15} \text{ m}$ . We know from high-energy scattering experiments,[29] on the other hand, that the electron interaction is point-like down to length scales below  $10^{-18} \text{ m}$ . Note that this does not necessarily imply that the electron is a point, but rather that the electron behaves in scattering events as though it is a single object with a  $1/r$  potential, without extra internal degrees of freedom. This is in contrast to the case for the proton, for example, where one has a set of sub-charges held together by binding forces. We can summarise these apparently contradictory elements as follows: the electron must have a *point-like interaction* and therefore must be a *single object*, but it must nevertheless have a *finite extent*.

Unless the electron has some internal structure, there is also some difficulty in reconciling the electromagnetic mass  $m = U_{elec}/c^2$  with the total momentum of its electromagnetic field.[26, 27, 28, 30, 31, 32] The problem can be resolved by postulating some internal forces, the so-called Poincaré-stresses, which, it has been argued,[30] will in any case have to be present to prevent the electron from flying apart due to the Coulomb interaction. As pointed out above, the presence of these binding forces between the electron charge distribution would interfere with the point-like interaction.

Another approach to solving the problems in the classical theory is to redefine the electromagnetic field energy and momentum in explicitly covariant form.[33, 34] If we want to

describe the internal structure of the electron, however, this has the serious drawback that, effectively, the electron's interior fields are normalised to zero.[35] Again, as is the case for quantum electrodynamics, one can say nothing about the internal structure of the electron.

In this paper, we formulate a simple model based on a single postulate: that there exists a state of a self-confined single-wavelength photon. We will argue that these distinct states are created and destroyed in charge-conjugate pairs. In effect we replace the postulated Poincaré- stresses, which confine the electron charge, with a postulated self- confinement mechanism for a photon. That we choose a photon rather than an electromagnetic wave introduces the relation  $E = \hbar\omega$ , and the model we propose will be semi-classical. The approach in what follows is to investigate the consequences of demanding periodic boundary conditions of length one wavelength on a quantised electromagnetic wave. We will see that this leads naturally to a model with a non-simply connected topology. The model suggests a possible origin for both charge and half-integer spin and also reconciles the apparently contradictory criteria discussed above for the electron. Although we have simply postulated that the photon may be confined, we will also discuss some possible origins for this confinement.

Our main motivation for the central postulate stated above arises from a consideration of the experimentally well-established (parapositronium) electron-positron annihilation and creation processes[36]



We have a time evolution of a state containing, on the one hand, two charged spin-half leptons, and on the other uncharged bosons of helicity one. If it were not for the different nature of the states on the left and right side, this reaction would, if taken alone, seem to suggest that the leptons and photons are in fact *different* states of the *same* object. Rather than hypothesising a new particle or field which would be the precursor of both the electron and the positron, we have tried to use the photon itself, which is the most obvious physical object with electric and magnetic field components which could give rise to the electron and positron charge and magnetic dipole.

## 2 The model

We envisage a quantised solution where, just as is the case for the free photon, we have time varying fields, but where the field distribution is self-confined in space.

The mass of any confined photon will be  $m = U/c^2$  where  $U = hc/\lambda$  is the energy of the photon of wavelength  $\lambda$ . From relation (2) it is clear that for the case where the electron and positron annihilate at rest, the decay photon wavelengths  $\lambda$  are just the electron Compton wavelength  $\lambda_C \equiv h/m_e c \approx 2.43 \times 10^{-12}$  m. We therefore, in the first instance,

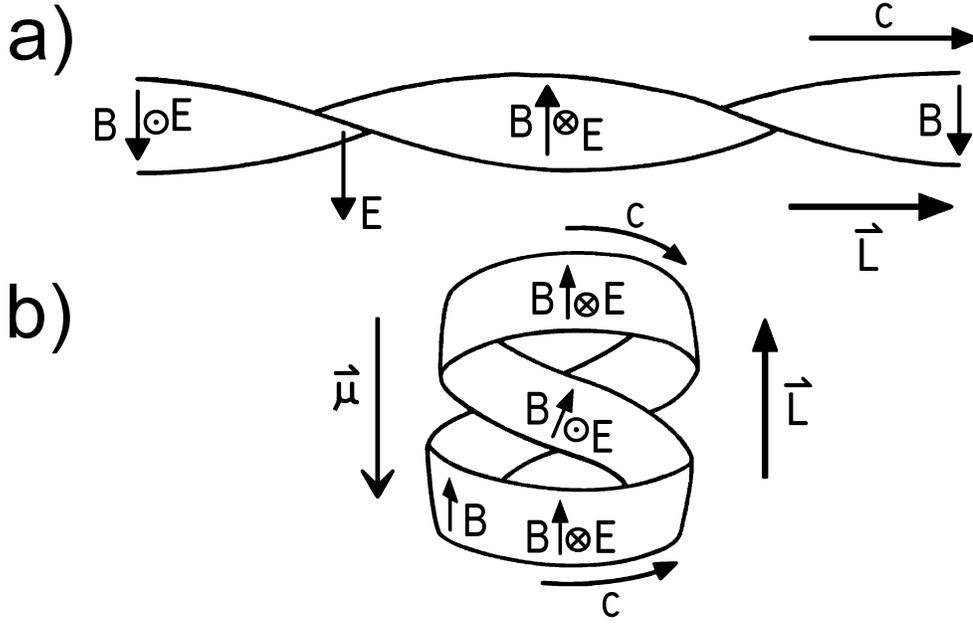


Figure 1: a) *Twisted strip model for one wavelength of a photon with circular polarisation in flat space. The  $\vec{B}$ -field is in the plane of the strip and the  $\vec{E}$ -field is perpendicular to it.* b) *A similar photon in a closed path in curved space with periodic boundary conditions of length  $\lambda_C$ . The  $\vec{E}$ -field vector is radial and directed inwards, and the  $\vec{B}$ -field is vertical. The magnetic moment  $\vec{\mu}$ , angular momentum  $\vec{L}$ , and direction of propagation with velocity  $c$  are also indicated.*

look for a quantised solution defined by periodic boundary conditions of length one Compton wavelength  $\lambda_C$ , which is confined to some closed path in 3-D space. Note that, insofar as photon propagation defines the shortest distance between two points (a geodesic), we may view our postulated confinement force as being equivalent to a closed, locally curved space. This curvature cannot arise from gravitation as in geometrodynamics[18, 19] as this is far too weak to replace a force of the magnitude of the Poincaré-stresses. The kind of curvature we are looking for need only apply to the self-confined photon, and will not necessarily affect any other object in the vicinity. We therefore envisage a solution more in terms of Maxwell's equations than in the theory of gravitation. We would like to emphasize that, even in the case of the linear Maxwell equations, localised solutions have been shown to exist,[14, 15, 16] though we think that nonlinear effects[20] must also play a role, as we will discuss later.

In looking for a plausible solution we demand that, along a local path element in this curved space, the photon is as similar as possible to a free-space photon. In particular, we have one of two orthogonal states with angular momentum  $\pm\hbar$ , corresponding to right or

left circular polarisation. In order to get some insight into the behaviour of a such a 3-D state, we model the circularly polarised photon as a twisted strip as illustrated in Fig. 1a. The strip represents an element of the physical photon and is used to visualise the evolution of its  $\vec{E}$  and  $\vec{B}$  vectors. The strip is distinct from the curved co-ordinate system in which the photon moves. The twist of the strip represents the rotation of the electric and magnetic field vectors. Here we show a single wavelength of one such state with the magnetic field vector  $\vec{B}$  in the plane of the strip and the electric field vector  $\vec{E}$  perpendicular to it.

Applying periodic boundary conditions of length one wavelength corresponds to bringing the ends of the strip together in such a way that there is still exactly one full twist in the resulting closed loop. The simplest possibility is the object illustrated in Fig. 1b. Only one of many possible similar paths is shown. This construction has the remarkable property of naturally forming a double loop, with one of the sides of the strip always facing outwards. Moving backwards or forwards along the strip represents a transformation in space, but equally well a transformation in time (i.e.  $x - ct$ ). This works for the photon illustrated in Fig. 1a, either moving along the strip, or waiting for the strip to pass you by, will give a rotation of the field components. Due to the locally curved space in Fig. 1b, however, this rotation is commensurate with the orbital rotation of the photon around the closed path. Movement in space now corresponds to moving along the axis of the twisted strip. The field still rotates, but so too does the direction of photon propagation, and these two effects combine in such a way that the electric field remains inward-directed. At the same time the magnetic field vector points upwards, as is clear from Fig. 1b. It is these properties, the inward-directed electric field and the upward directed magnetic field which, as we will discuss in what follows, give rise to a charge and a magnetic moment. We would like to emphasize that the photon remains uncharged. It is the confinement, the topology, and the commensurability of the field components with the orbital path which are important. It is crucial that there is exactly one full twist, a half twist or double twist, for example, would not give rise to a charge. Depending on the sense of path closure (whether the ends of the strip have been brought together into or out of the plane of the paper), we see that outside the object the resultant electric field vector always points inwards (electron- like) or outwards (positron-like). In a creation process both these sorts must be present in pairs to ensure the conservation of charge, four-momentum and angular momentum.

The special case where both loops lie on top of one another is particularly important. The loop radius is then exactly  $\lambda_C/4\pi$ , and this scale of length is intrinsic to our model. The circulation repeats itself with a period of half a wavelength. In flat space, this would lead to total destructive interference everywhere along the path. Within our object, where we have demanded that space is curved, the interference is always constructive as is clear from Fig. 1b. Although the electric field vector is always inward directed, it still undergoes a local rotation around the path direction of  $2\pi$  on circulating twice around the loop, just

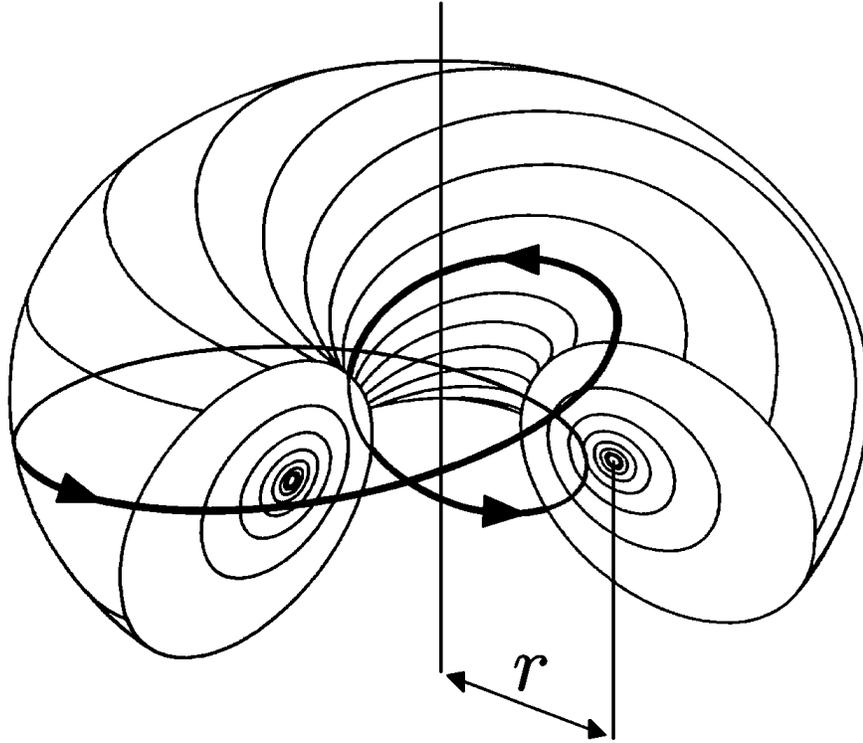


Figure 2: *Schematic of the internal energy flow in the model. The lines of flow (geodesics) circulate twice around a family of nested toroidal surfaces before closing on themselves. The left-handed case is illustrated. For clarity, one complete double-loop path is emphasised. The toroidal structure is characterised by a length  $r = \lambda_C/4\pi$ .*

as is the case for a single wavelength of a free-space photon. Note that we do not have a standing wave, but a stationary wave propagating around a double loop. Hence, this state will have angular momentum. We will come back to this in the section on the spin.

In Fig. 1b we have considered one possible path which fulfils the boundary conditions. Since we talk about a single photon confined in a region of the order of its own wavelength we are firmly in the diffractive limit, where it makes little sense to talk about a specific photon “path”. Any path which fulfils the constraint of having length  $\lambda_C$  may contribute to the structure of the complete object. Further, a self-consistent set of paths must not cross each other. In Fig. 2 we illustrate the geodesics (corresponding to the streamlines of energy flow) for a set of possible paths fulfilling these conditions. In bold we illustrate one of the particular double-looped paths (similar to Fig. 1b). From this we see that the paths circulate twice around a toroidal surface before closing on themselves. It is clear from

the diagram that the mean radius of energy transport (the eye of the torus) is close to the intrinsic scale of length in our model ( $\lambda_C/4\pi$ ), corresponding to the case where both the loops lie on top of one another. We would like to emphasize that it is possible to devise many topologies (knots) which are consistent with our initial postulate, but the toroidal topology which arises from the double loop model is the simplest and most natural of these.

To proceed further, we also need to estimate the total size of our object, which is considerably larger than the intrinsic length scale ( $\lambda_C/4\pi$ ), as is clear from Fig. 2. An upper limit on the size of the object is obtained by considering the extremal paths which also fulfil the condition that they have length  $\lambda_C$ . These paths, which correspond to the limiting case where the confined photon travels radially outwards and inwards, have maximal radius  $\lambda_C/2$ . The limitation of the speed of light means that only paths within this radius can provide a contribution to the circulation of energy inside the object. This radius constitutes a “rotation horizon” and sets an upper limit on the size of the object equal to a Compton wavelength. At the same time, we can estimate a lower limit on the size from the fact that it is impossible to confine an arbitrary wave into a box which is smaller than one half of its wavelength. Moreover, the minimum diameter of, for example, a spherically symmetric dielectric cavity is equal to a full wavelength. These conditions constrain the effective size of our self-confined object to be close to or equal to  $\lambda_C$ . For simplicity in the above argument, we have assumed that the relative dielectric constant and magnetic permeability are unity everywhere throughout the volume contained within the rotation horizon.

The rotation horizon forms a boundary between an inside region where space is curved, at least for the spinning photon, and an outside region where space is relatively flat. In this outside region there must also exist an electric and magnetic field in order to fulfil the conditions of electromagnetic continuity at the boundary. These external fields constitute the apparent charge and magnetic dipole. The external field is non-rotational and will slightly reduce the energy and hence shift the wavelength and frequency of the spinning photon inside the boundary. We write the internal wavelength as  $\lambda = a\lambda_C$ . As we will show, this correction is small and is of the order of  $\alpha/2\pi$  where  $\alpha \approx 1/137$  is the fine-structure constant. Although we have argued that, internal to our model, we have a toroidal topology, it is not obvious what the precise form of the rotation horizon should be. Clearly, although the electric field is radial at large distances, the physical electron itself cannot be perfectly spherically symmetric since it has a magnetic moment. For simplicity, however, we assume that we have a photon confined in a spherical cavity of diameter *exactly*  $\lambda$ .

Our model has some striking analogies with the detailed solution of the electron motion calculated by Dirac.[37] He has shown that one can analyse the electron motion into a part that describes the (relativistic) motion of the electron as a whole plus a light-speed oscillatory part of twice the Compton frequency  $2\omega_C$ , the so-called “Zitterbewegung”.[38] Further, any instantaneous measurement of any component of the electron velocity will

always yield one of the eigenvalues of  $\pm c$ . This “Zitterbewegung”, the frequency  $2\omega_C$ , and the instantaneous velocity eigenvalues of  $\pm c$  are clearly also features of our double-looped confined photon. This correspondence becomes even more clear in the geometrical “Zitterbewegung interpretation of Quantum Mechanics” developed recently by Hestenes.[39] In this work it is shown that the Zitterbewegung may be used to interpret the half-integral electron spin, and that the trajectory of a moving Dirac electron may be viewed as series of light-like helices of radius  $\lambda_C/4\pi$  defined by the rotation of the electron energy-momentum flux in a plane perpendicular to the spin.[40, 41] This is just the trajectory of the eye of the torus for our model (Fig. 2). Our model is seen to be very close to this geometrical interpretation of the Dirac electron, except that we have arrived at this solution from a postulated (photon) self-interaction, and the Zitterbewegung is that of the electromagnetic field of a confined photon rather than that of an electron wavefunction.

### 3 The charge

Note that in electron-positron pair creation (relation (2)) the total electric field divergence of the system remains zero, in accordance with the conservation of charge. This is also the case for our model, where equal and opposite localised field divergences are created in pairs.

In Fig. 1b the electric field is always inward (outward) directed because the photon orbital rotation and the photon field rotation are commensurate. Local to either the “electron” or the “positron”, we have a non simply-connected (toroidal) topology embedded in a simply-connected space. This leads to a so called topological charge.[42] In our case, since we have an electromagnetic field present in this topology, this may also give rise to a real electric charge.[14, 15] Since we only have a phenomenological model we will not prove rigorously that  $\nabla \cdot \vec{E} \neq 0$  for an outside observer. Assuming that the fields can indeed be folded in such a way, so that we have two equal and opposite charges, we can estimate the charge magnitude from the field magnitude close to each object.

The magnitude of the apparent charge of our model object is based on the length scales estimated in the previous section. We confine an arbitrary photon with wavelength  $\lambda$  to a spherical volume  $V = \frac{4}{3}\pi(\lambda/2)^3$ . The energy density of the electromagnetic field in the volume is  $W = \frac{1}{2}(\varepsilon_0|\vec{E}|^2 + \mu_0^{-1}|\vec{B}|^2)$ . For a propagating photon inside the volume, where space is curved, we take  $E = cB$  and  $c^{-2} = \varepsilon_0\mu_0$  as is the case for a free-space photon. The electric field energy  $U_E$  and the magnetic field energy  $U_B$  are then one half of the total confined photon energy  $U$  (i.e.  $U_E = U_B = \frac{1}{2}U$ ). We find for the average energy density of the electric field in the volume  $V$ ,  $W_E = U_E/V = \frac{1}{2}U/V$  and also  $W_E = \frac{1}{2}\varepsilon_0 E^2$ . The average magnitude of the electric field inside the model electron is then

$$\langle E \rangle = \sqrt{\frac{6hc}{\pi\varepsilon_0\lambda^4}}, \quad (3)$$

To estimate the charge in our model we need to compare the magnitude of the inward directed electric field to that for a point charge at the origin. Making the plausible assumptions that the relevant length scale from where the electric field is effectively inward-directed is the mean radius of energy transport  $\bar{r} = \lambda/4\pi$ , and that the average electric field of the confined photon, Eq. (3), is a good estimate of the field at this radius, we obtain the effective charge,  $q$ , by comparing this to the Coulomb field of a point charge at distance  $r = \bar{r}$

$$E = \frac{q}{4\pi\epsilon_0 r^2}, \quad (4)$$

which then yields the charge from our model in terms of the elementary charge  $e$

$$q = \frac{1}{2\pi} \sqrt{3\epsilon_0 \hbar c} \approx 0.91e, \quad (5)$$

where this apparent charge arises from the electric field of the confined photon. Note that  $q$  is *independent* of the energy of the photon (the size of the object) and is a result of the toroidal topology. It depends on the detailed distribution of the internal fields and also on the precise value we choose for the effective charge radius. Note, however, that any reasonable variation of these parameters will still yield a finite value close to that of the elementary charge. Here, we have made the simple assumptions that the field distribution within the object is homogeneous, and that the relevant transport radius is that of the toroid illustrated in Fig. 2. For these assumptions  $q$  is remarkably close to the elementary charge.

## 4 Spin

The existence of half-integral spin is intimately connected with relativistic quantum mechanics; it has no correspondence in classical mechanics.[37, 43] Although it is well known that orbital angular momenta are usually integral, this need not necessarily be the case in a non-simply connected topology. In this section we will show that, at least for one special direction (the  $z$ -axis in Fig. 2), the spin in our model is  $\pm\frac{1}{2}\hbar$  as a result of the non-Euclidean topology of our model.

The configuration space of the half-integral quantum spin is quite different from that of a spinning rigid body. The quantum mechanical commutation relations allow only the total angular momentum squared  $L^2$  and one spatial component, say  $L_z$ , to be measured simultaneously, and for spin one half any measurement will yield the values  $L^2 = \frac{3}{4}\hbar^2$  and  $L_z = \pm\frac{1}{2}\hbar$  respectively. This behaviour cannot be modeled by a rotating rigid body since it has effectively only one rotation axis, which will be preserved if no external force acts. Also, for a rigid body, the magnitude of the spin measured will depend on the projection on the measurement axis. In contrast to the classical case, in quantum mechanics there is no

pre-defined direction to the spin axis. These properties provide a formidable challenge to any mechanical model of the structure of the electron. Despite these obvious difficulties it has proved possible to devise classical models which share the projection and commutation properties of half-integral spin.[44, 45] Certain solitons in toroidal coordinates have been shown to be classical analogs of fermions.[49, 50, 51, 52] There are strong analogies between the transformation properties of the classical electromagnetic field and spinors.[46, 47] This point has been discussed extensively by Kramers.[48] In this paper we will not extract the full quantum mechanical projection properties or statistics of spin from our semi-classical model but a subset of these properties which, at least, goes beyond that possible with, for example, a charged rigid body.

The rotational energy of a relativistic object is  $U_{rot} = L\omega$ , with  $L$  the angular momentum, and  $\omega$  the angular frequency. For a photon  $L = \hbar$ , and the total energy of a photon with frequency  $\omega$  is  $U_{photon} = \hbar\omega$ . Thus, the energy of a photon is entirely electromagnetic and contained in its spin. The confined photon in our model has to travel around *twice* to complete its path of length  $\lambda = 2\pi c/\omega$ . Consequently, the internal rotational frequency of the model is twice the photon frequency  $\omega_s = 2\omega$ . The internal rotational energy is equal to the confined photon energy, and we may write  $U_{model} = L\omega_s = \hbar\omega$ . Our model must then have an *intrinsic* angular momentum  $L = \hbar\omega/\omega_s = \frac{1}{2}\hbar$ . We see that this describes an object of half-integer spin. If the spin-statistics theorem applies, our self-confined photon should be a fermion. This is again a direct consequence of the topology of our model; the field vectors must rotate through  $720^\circ$  before coming back to their starting position with the same orientation. In quantum mechanics, the spin angular momentum has a fixed value  $s = \frac{1}{2}$ , therefore we cannot take the intrinsic spin to a classical limit by letting  $s \rightarrow \infty$  and there is no classical correspondence with half-integer spin. In our model this is ensured because, for our topology, we have necessarily one and only one wavelength, and this gives a fixed, length-scale independent value of  $s$ .

We now look more closely at the internal dynamics of our model, as illustrated in Fig. 2. Consider the orbital angular momentum of the localised photon at the mean energy transport radius ( $\bar{r} = \lambda/4\pi$ ). This is

$$L_{orbit} = |\bar{r} \times \vec{p}| = \frac{\lambda}{4\pi} \frac{U_{photon}}{c} = \frac{\hbar}{2}, \quad (6)$$

Here the factor of  $\frac{1}{2}$  arises because the photon, as observed from outside, traverses a *double* loop within the toroidal topology before returning to its original position with the same orientation. This orbital spin is a pre-existing vector with a definite direction, but this is not the whole story. Define a co-ordinate system with the  $z$ -axis the orbital spin axis passing through the center of the torus. At any instant an element of field will be rotating both around this axis, and about a perpendicular axis tangential to the eye of the torus and located in the  $x, y$ -plane (see Fig. 3). This tangential spin axis is always perpendicular to

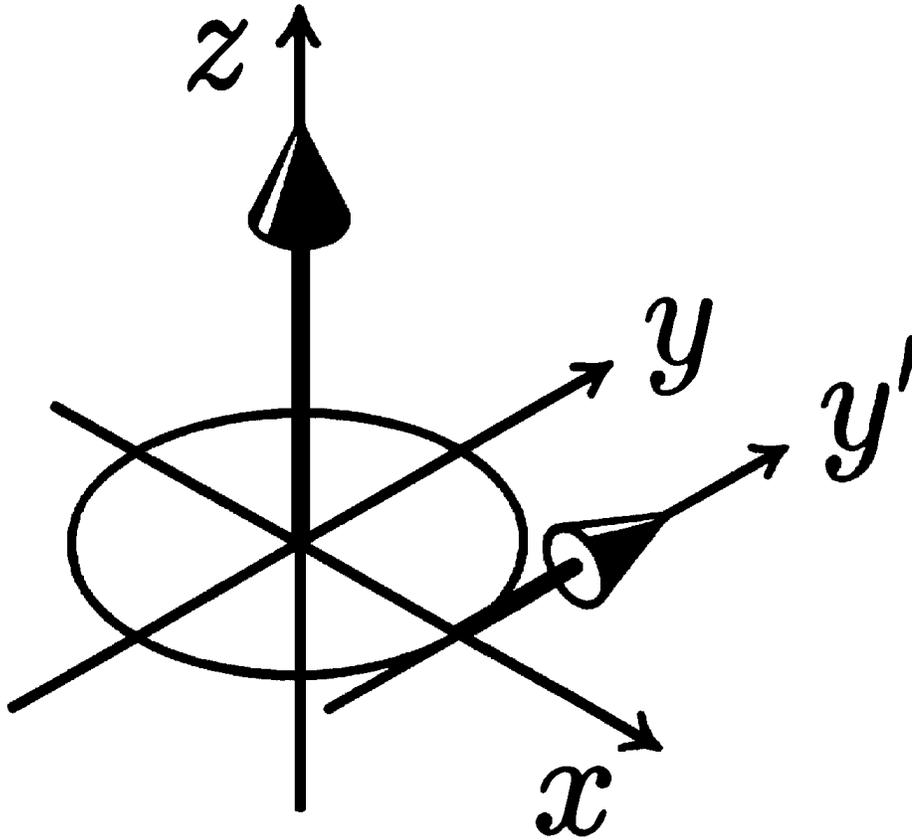


Figure 3: *Definition of a co-ordinate system with the  $z$ -axis passing through the center of the torus. At any instant an element of field will be rotating both around the  $z$ -axis (orbital spin), and about a perpendicular axis tangential to the eye of the torus (intrinsic photon angular momentum).  
The tangential spin-axis performs a rotation in the  $x, y$ -plane about the  $z$ -axis with frequency  $2\omega_C$  and radius  $\lambda_C/4\pi$ .*

$z$  but performs a rotation in the  $x, y$ -plane about the  $z$ -axis with frequency  $2\omega_C$  and radius  $\lambda_C/4\pi$ . We have a rotation about  $z$  combined with an alternating rotation about  $x'$  and  $y'$ , and hence effectively have three rotation axes while we still have just one object. The projection on  $z$  of the tangential spin axis is always zero and therefore the  $z$ -component of the angular momentum will remain the sharp value of  $L_z = \pm\frac{1}{2}\hbar$  we have calculated for the orbital angular momentum. Clearly, although we have a single object, the motion is not describable by a rigid-body rotation because the two rotation axes are separated from each other by  $\lambda_C/4\pi$ , so that it is not even possible to define a single effective instantaneous rotation axis. The configuration space of our model is already quite different from that of a classical top. Further, the rotation depends on the (unobservable) phase of the confined photon. Just as is the case in quantum mechanics there is no pre-existing spin axis. The projection on the internal  $z$ -axis, however, is a constant of the motion and remains  $\pm\frac{1}{2}\hbar$ . For our object to be an electron the average angular momentum around both  $x$  and  $y$  should be also  $\pm\frac{1}{2}\hbar$  because then the spin magnitude squared  $L^2 = L_x^2 + L_y^2 + L_z^2$  would have the quantum mechanical value of  $\ell(\ell+1)\hbar^2$ . We can see no simple argument, however, as to why the intrinsic photon angular momentum should distribute itself in this way. It is interesting to note, however, that the relativistic orbital velocity of the bound photon gives rise to an apparent rotation of the photon frame as seen by an external observer. This is essentially the same effect as that, for example, leading to the Thomas precession of an electron orbiting about a nucleus. It is easy to show that this leads to a counter-rotation in the  $x, y$ -plane of the internal frame of the photon with the same magnitude as, but with opposite sense to the orbital rotation. This means that any initial spin distribution will appear to remain fixed.[53]

The angular momentum component corresponding to the intrinsic photon angular momentum about the eye of the torus (see Fig. 2) constitutes an extra internal degree of freedom in our model. This has only two possible values corresponding to a right or left circularly polarised photon. We interpret these two possible states as corresponding to the SU(2) of electron spin. The flow around the central axis of the torus is always left-handed (right-handed for the positron-like case) with respect to the magnetic-moment direction. This is a natural consequence of having a right-handed set for the photon co-ordinate system  $\vec{E}$ ,  $\vec{B}$  and  $\vec{c}$ . Note, however, that the mirror image of an electron will also be an electron but in the other spin state.

In conclusion there is an extra internal vector in our model corresponding to the two states of the photon spin which leads to an SU(2) symmetry, the rotation of the object as a whole is not describable as any rigid-body rotation and, in at least one direction, the angular momentum is half-integral.

## 5 The magnetic dipole moment

The total energy contained in the electric field outside the rotation horizon with radius  $r_{hor} = a\lambda_C/2$  is

$$U_{E,ext} = \int_{|\vec{r}| \geq r_{hor}} W_E d\vec{r} = \int_{r_{hor}}^{\infty} \frac{q^2}{8\pi\epsilon_0 r^2} dr = \frac{\alpha'}{2\pi a} \hbar\omega_C, \quad (7)$$

with  $\alpha'$  the “fine-structure constant” for our model, defined as

$$\alpha' \equiv \frac{q^2}{4\pi\epsilon_0 \hbar c} = \left(\frac{q}{e}\right)^2 \alpha, \quad (8)$$

and  $a = \lambda/\lambda_C$ . As already discussed, the external electric field is non-rotational. The total energy in the object  $U_{model}$  is the sum of the energy in this non-rotating external part  $U_{ext} = U_{model}\alpha'/2\pi a$  and the internal part  $U_{int} = U_{model}(1 - \alpha'/2\pi a) \approx 0.999U_{model}$ , which contains all the rotational energy of the body. This means that the effective frequency  $\omega = U_{int}/\hbar$  of the confined photon is slightly smaller than the Compton frequency, and that the size of the rotation horizon, and also of the energy transport radius, has to be adjusted accordingly. So we have  $\frac{1}{2}a\omega_s = a\omega = \omega_C = U_{model}/\hbar$ , and  $U_{int} = \frac{1}{2}\hbar\omega_s$ . Hence we find that  $a = 1 + \alpha'/2\pi$  which gives a corrected value of the rotation horizon of  $(1 + \alpha'/2\pi)\lambda_C/2$ . As stated previously this correction has no effect on either the charge or the intrinsic spin in our model. Note that there is also some energy in the external magnetic dipole field in our model, which is, however, two orders of magnitude less than for the electric field.

In the same way that we compared the electric field  $\vec{E}$  of the photon with the Coulomb field to obtain the charge in Eq. (5), we now compare  $\vec{B}$  with the field of a magnetic point dipole with strength  $\mu_d$ . The components of the field are given by

$$B_r = \frac{2\mu_0\mu_d \cos\theta}{4\pi r^3}, \quad (9)$$

$$B_\theta = \frac{\mu_0\mu_d \sin\theta}{4\pi r^3}, \quad (10)$$

$$B_\varphi = 0. \quad (11)$$

We assume again that the confined photon has the properties  $E = cB$  and  $\vec{E} \perp \vec{B}$ , so that in the equatorial plane ( $\theta = \frac{1}{2}\pi$ ) of our model we have  $E_r = cB_\theta$ . Using Eq. (4) and (10), it now follows directly that  $q/\epsilon_0 = \mu_0\mu_d c/r$ . Hence, we find  $\mu_d = qrc$ . Taking, as in the calculation of the charge,  $r = \bar{r} = a\lambda_C/4\pi$  with  $\lambda_C \equiv h/m_e c$ , we find

$$\mu_d = \left(1 + \frac{\alpha'}{2\pi}\right) \frac{q\hbar}{2m_e} = sg\mu_q, \quad (12)$$

with  $s = \frac{1}{2}$  the spin quantum number,  $g = 2a = 2(1 + \alpha'/2\pi)$  the gyromagnetic ratio, and  $\mu_q$  the magneton for an object with charge  $q$  and mass  $m_e$ . Apparently, our model has very

nearly *the same anomaly* in  $g$  as the electron and muon. In quantum electrodynamics, it follows from (first order) radiative corrections[54, 55, 56] that

$$g = 2 \left( 1 + \frac{\alpha}{2\pi} \right) \approx 2.0023 . \quad (13)$$

In our model, the anomaly in  $g$  originates from the fact that a small fraction of the mass (energy) does not circulate within the body, but appears as a non-rotational external field.

## 6 Point-like interaction and the “harmony of phases”

That our model is purely electromagnetic rather than material has some interesting consequences when it is viewed from a frame in relative motion.

When two spinning bodies collide, the outcome often depends on how they are spinning. For example, the path taken by an electron after a collision can be affected by its spin, just as for a spinning billiard ball. It is clear that the influence of the spin on a billiard balls trajectory depends on the *ratio* of the rotational and kinetic energy of the ball. The influence of the spin in a collision should decline as the kinetic energy of a ball, with given spin, is increased. At a sufficiently high collision energy it should make no difference whether two colliding billiard balls are spinning the same way or in opposite directions.

In this argument, we have made one important assumption implicitly, that the rotational energy of the balls is not affected by a *linear* acceleration. This is reasonable if the accelerating force acts on the center of mass of the ball, which coincides with the axis of rotation. There is then no torque to spin-up the ball. The situation is quite different for photons. If we blue-shift a circularly polarised photon in, for example, a gravitational field, its frequency, and hence its rotational energy increases, whilst its angular momentum remains constant. Equivalently, if we transform to a frame moving parallel but opposite to the photon momentum, so that there is a Lorentz-contraction in the direction of motion, the angular momentum  $L = \hbar$  will be conserved, but again the rotational energy  $U = \hbar\omega$  will be larger due to the blue shift of the frequency. We now investigate what happens in the case of a localised photon. The Doppler shift of a free photon with angular frequency  $\omega$  as emitted from a source moving with velocity  $\vec{v}$  with respect to some frame of reference is given by

$$\hat{\omega} = \gamma(\omega + \vec{v} \cdot \vec{k}) = \gamma\omega \left( 1 + \frac{v}{c} \cos \theta \right) , \quad (14)$$

with  $\theta$  the angle, in the frame of the source, between the wave vector of the photon  $\vec{k}$  and the velocity  $\vec{v}$ , and

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} . \quad (15)$$

We now consider a confined photon, going through some arbitrary closed loop of length  $S = \lambda$ , with an orbital frequency  $\omega_{orb} = \omega$  equal to the frequency of the internal photon.

The path shown in Fig. 4a is a special case of this. If the object moves with speed  $\vec{v}$ , the internal photon will be alternately red and blue shifted during a single cycle (see Fig. 4b). To find the total energy in this (moving) object, we integrate the instantaneous internal frequency  $\hat{\omega}$ , given by Eq. (14), over the *closed* path  $S$ , which just gives the mean energy over one oscillation period of the internal photon:

$$U = \frac{\hbar}{S} \oint_S \hat{\omega} ds = \hbar\gamma\omega = \hbar\langle\hat{\omega}\rangle, \quad (16)$$

where  $\langle\hat{\omega}\rangle = \gamma\omega$  is the mean internal frequency of the photon, and we have used the fact that  $\vec{v}$  represents a conservative vector field (a Lorentz frame). This leads to  $U = \gamma U_0$ , and thus, since  $U = mc^2$ , that  $m = \gamma m_0$  as for any massive particle. We can interpret the increase of the mass of the object as an increase of the *internal rotational energy*. The angular momentum  $L$  of the object is conserved instantaneously, so the rotational energy  $U = L\langle\hat{\omega}\rangle$  increases with  $\langle\hat{\omega}\rangle$ . This implies that the effective size of the object must reduce when viewed from a moving frame since  $L = |\vec{r} \times \vec{p}|$  with  $p = U/c$  implies that the mean radius of the field energy scales with  $\langle\hat{\omega}\rangle^{-1}$ . Although the photon flux is conserved, the energy-momentum flux will be larger in the blue shifted part than in the red shifted part, as must be the case if there is to be a net transport in the direction of motion.

In the proper frame of our model (the rest frame), both the internal photon and its orbital rotation have an eigenfrequency  $\omega$  which is equal to the Compton frequency  $\omega_C$ . In a moving frame, however, they diverge: the mean internal frequency of the photon  $\langle\hat{\omega}\rangle = \gamma\omega$  will increase; simultaneously, the orbital frequency  $\omega_{orb} = \omega/\gamma$  of the photon will decrease

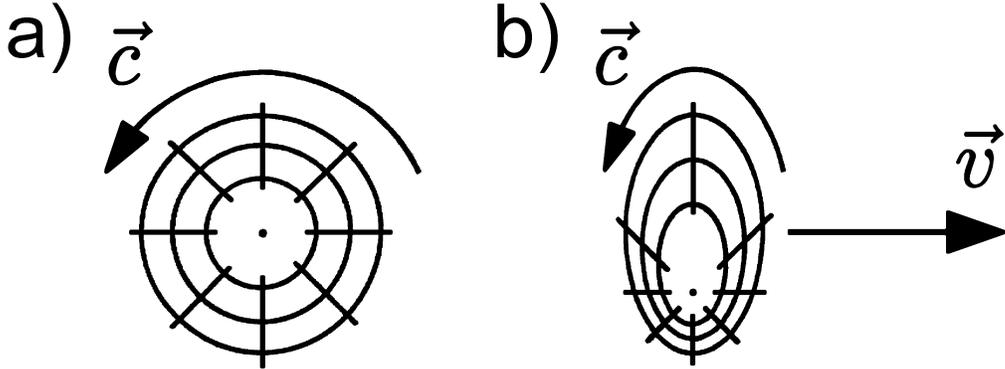


Figure 4: Schematic diagram of a localised-photon model: **a)** at rest (the proper frame), and **b)** moving at speed  $\vec{v}$ , where the object as a whole is Lorentz contracted in the direction of motion and the circulating photon is alternately red and blue shifted. The ticks mark equal intervals in phase, and the line density represents the momentum density.

due to the relativistic law of the slowing down of clocks. Despite the difference in frequency, at any point in space-time these two oscillations must still be in phase, just as they are in the proper frame. This provides a possible physical origin for the postulated law of the “harmony of phases” first proposed by de Broglie,[57, 58] which lies at the origin of quantum mechanics. We now discuss the consequences of the above in more detail.

In the proper frame, the phase of both the orbital rotation and the internal photon is  $\phi = \omega_C t_0$ . In a frame where the object is moving with velocity  $\vec{v}$  in the  $x$ -direction, the phase of the orbital rotation will be given by  $\phi = \omega_{orb} t$ , where the time  $t$  is linked with the proper time  $t_0$  by the relation

$$t_0 = \gamma \left( t - \frac{vx}{c^2} \right) , \quad (17)$$

and hence the phase of the internal photon in this frame can also be written as

$$\phi = \omega_C t_0 = \omega_C \gamma \left( t - \frac{vx}{c^2} \right) . \quad (18)$$

This equation can be interpreted as describing a wave of *higher* frequency  $\gamma\omega_C = \langle \hat{\omega} \rangle$ , which is propagating along the  $x$ -axis with phase-velocity

$$v_p = \frac{c^2}{v} , \quad (19)$$

where the velocity of the particle  $v$  has the role of the “group velocity”. The distance in space between two consecutive wave crests is then

$$\lambda_B = \frac{2\pi}{\langle \hat{\omega} \rangle} v_p = \frac{2\pi c^2}{\gamma v \omega_C} = \frac{h}{\gamma m_0 v} , \quad (20)$$

which is just the de Broglie wavelength. This extra oscillation arises as a direct result of having a confined light-speed wave. Note that using  $x = vt$  in Eq. (18) it is easy to see that  $\omega_{orb} t = \omega_C t_0$  and, indeed the orbital and internal photon oscillations are in phase. Now we may write for a moving frame

$$\hbar \langle \hat{\omega} \rangle = \gamma \hbar \omega_C = U = \sqrt{U_0^2 + p^2 c^2} = \hbar \sqrt{\omega_C^2 + \omega_B^2} , \quad (21)$$

from which we see that the total energy may be expressed in terms of the time-like oscillation frequency of the localised photon  $\omega_C$  defined in the proper frame of the object and, as a consequence of the relativistic transformation of this, an additional space-like oscillation frequency  $\omega_B = 2\pi c / \lambda_B$ .

It is well known that in high-energy physics scattering experiments the interaction between two electrons remains point-like down to length scales of the order of  $10^{-18}$  m. The size of our object is much larger than this, being of the order of  $10^{-12}$  m. Despite this, the internal structure of such a self-confined photon will not be resolved regardless of the scattering energy, as we now show.

Provided that the photon is the only constituent of our object, i.e. that it indeed confines itself, all of the four-momentum will be carried by the photon. In high energy collisions no internal structure will be resolved since there is no extra particle or field to absorb the excess four-momentum. In a head-on collision between two confined photons, the interaction will be essentially between the blue-shifted regions (see Fig. 4) which are converging and which have a characteristic size  $\lambda_C/(4\pi\gamma)$ , and not with the red-shifted parts which are still diverging from each other at the speed of light. The maximum resolving power of one object for the other is just  $\lambda_B/2$  in their centre of mass system. Using  $\lambda_C \equiv h/(m_0c)$  in Eq. (20), we find the relation

$$\gamma^{-1}\lambda_C = \frac{v}{c}\lambda_B . \quad (22)$$

Now the characteristic size of our object  $\lambda_C/(4\pi\gamma) < \lambda_B/2$  and thus the internal structure of neither of the objects is revealed, regardless of their scattering energy. In high energy collisions, therefore, the interaction with this object will scale with the energy and will remain point-like. Note that this behaviour is a consequence of the fact that our model is purely electromagnetic and hence, at relativistic energies, scales in effective size in exactly the same way as a free-space photon.

In our picture, the electron remains a single elementary particle, but that elementary particle is now a new state of the photon. Our model therefore fulfils the three conditions mentioned in the introduction. Namely that we have a single object of finite extent, but with a point-like interaction.

We have argued that in our model all the energy is electromagnetic and contained in the spin, regardless of its state of motion. In principle strong spin effects should play a role for polarised electron-electron scattering. As we have discussed above, however, it is not clear that electron overlap will occur, even for very high-energy scattering. In spin-polarised proton scattering, however, the spin has been shown to have an unexpectedly large effect.[59, 60] The cross-section for spin-parallel proton-proton scattering at high four-momentum transfers was found to be several times larger than that for spin-antiparallel scattering. We expect polarised particle scattering to provide a route for the experimental testing of our model.

## 7 Confinement

As discussed in the introduction, the confinement of the electron charge is still an open question. Casimir has proposed that a charged conducting shell may be bound by the vacuum fluctuations.[61] This has been shown to be unstable for a spherical shell[62, 63] but may be stable for flatter systems such as an oblate spheroid or a torus.[64] This model has the advantage that the ‘‘Poincaré-stresses’’ are themselves electromagnetic, but leaves open the question of the nature of the shell.

For our model, we would like to be able to write down a set of equations, similar to Maxwell's equations, which describe the self-generation of the confined photon from its constituent fields. We do not have such a detailed dynamical confinement scheme, however, the model allows us to discuss some possible confinement mechanisms in a qualitative way. The photon must confine *itself* since if any other particles or fields were present, as is the case in, for example the proton, then this would affect the point-like interaction between electrons and photons observed in experiment. Although, as mentioned previously, circulating solutions of the linear Maxwell equations have been shown to exist,[14, 15, 16] the fact that the electron does not have arbitrary mass means that some extra, presumably non-linear, effect must also play a role. It is well known that there may be solitary-wave solutions in a nonlinear medium. The possibility that such disturbances may exhibit particle-like behaviour (solitons) has been recognised for some time,[6, 7, 52, 65, 66, 67, 68] and it has been shown that at least some of these objects also fulfil the “harmony of phases” discussed in the previous section.[69]

We have already noted that the curvature of space due to the mass of our object is far too weak to confine a photon. It has been argued, however, that an intense electromagnetic energy density may change the curvature of space-time, independent of the effect of gravitation. This may be interpreted as being equivalent to a nonlinear vacuum.[20] It is well known that the vacuum must be nonlinear, as is illustrated by the simple fact that pair creation can occur only above a certain threshold. As one confines successively shorter wavelength photons into a volume of diameter  $\lambda$ , the average energy density increases as  $1/\lambda^4$ , whereas the energy (and hence the mass) increases only as  $1/\lambda$ . The existence of photons with energies much larger than the electron mass, however, implies that something more is required than simply a high energy density to create an effective local curvature of space. It is known that in the vacuum, non-linear effects can play a role if  $\vec{E}$  and  $\vec{B}$  are not perpendicular, or if  $E \neq cB$ . This is for example the case in strongly focused laser beams, or plane waves in the presence of external fields.[55, 56] Consider the process where two identical photons (i.e. with the same helicity) collide head-on in the reference frame where both have the same energy. The interference of two counter-propagating circularly-polarised electromagnetic waves has been discussed extensively in the literature [70, 71, 72], and gives rise to a twisted-mode standing wave solution to Maxwell's equations where  $\vec{B}$  is everywhere and at every instant parallel to  $\vec{E}$ . This means that the Poynting vector must be identically zero everywhere (and for all time), and that this mode is everywhere non-propagating. It is just this case, with the extra condition that we have two photons with a wavelength equal to or shorter than the electron Compton wavelength, where pair creation can occur. We would expect that the vacuum nonlinearity could best be studied in an experimental configuration of this kind, where identical helicity photons travel in opposite directions[71]. In our model, we view the pair creation process as arising from such a non-propagating intermediate state

with the required curved space-time. This state may then decay back into two photons or into an electron-positron pair. It is important to note that if a confined photon state with toroidal topology is formed, it will be stable provided that it is the lightest such object with this topology.[15, 49, 73, 74]

## 8 Other particles

If the electron is indeed constituted by a photon, other elementary particles may also be composed of photon states, but in some other configuration to that shown in Fig. 2. The possibility that muons and tauons may be formed by electron-like states with a different internal curvature has been discussed in the literature.[39] We speculate that the hadrons may be described by composite confined photon states. If we identify a quark with a confined photon state which is not sufficient in itself to complete a closed loop in space, but transforms a photon going in one spatial direction to one travelling in another, it would then only be possible to build closed three-dimensional loops from these elements with  $qqq$  and  $q\bar{q}$  combinations.

Until now we have discussed the possibility that a bound photon state may give rise to an electric monopole. Equally, we could have required that the magnetic field in Fig. 1b was always inward-directed and the electric field always upwards, giving rise to an object with a magnetic monopole and an electric dipole.[75] The only difference between these two cases in our model is an internal  $90^\circ$  rotation of the fields around the eye of the torus. Dirac notes that the magnetic monopole field strength is  $1/(2\alpha)$  times larger than the electric monopole field strength.[76] It is impossible to construct such a large field strength in our model because the mean radius of energy transport would have to be outside the rotation horizon. We conclude that, given the Dirac result, it is not possible to form a stable magnetic monopole within the framework of our model.

## 9 Conclusions

The primary reason that the electron is considered to be elementary is that experimentally it appears to be point-like and hence structureless. At the same time we are confronted with the fact that it has a rich set of properties which are fundamental to its nature. It has an elementary charge, a half-integral spin, a definite mass, a well defined dipole moment, an anomalous spin factor  $g - 2$  and of course a wave-particle nature. It seems inappropriate to think about such things as the elementary charge as a separate building block from the elementary particle which carries it. A deeper understanding requires a unification of the aspects discussed above in terms of an underlying principle.

In our model, the point-like interaction observed in high-energy electron scattering experiments is ensured in that one, and only one, constituent is present, in this case a single photon, and that its size scales with the total energy. Although we have not addressed the absolute scale of length in our model, and hence have not fixed the electron mass, the characteristic size is finite, and is related to the Compton wavelength in a simple way. The structure is therefore that of a single object of finite extent, with presumably a finite self-energy. A charge arises in our model from the topology of the photon path, in combination with the photon electric field. Although we have not discussed the origin of the quantisation of charge, the value we obtain is at least independent of the length (mass) scale. We have discussed the connection between half-integral spin and the topology of our object and conclude that our model has at least some of the properties of a fermion. An anomalous spin factor,  $g - 2$ , arises directly from a consideration of the energy in the external field in our model and is identical to that calculated for the electron in first-order quantum electrodynamics. Our model gives a physical origin for the postulate of the “harmony of phases” proposed by De Broglie which lies at the heart of quantum mechanics. The model excludes the possibility of a magnetic monopole.

In this paper we have argued that the single extra postulate: that there exists a confined single-wavelength photon state, leads to a model with non-trivial topology which allows a surprising number of the fundamental properties of the electron to be described within a single framework.

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